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MULTICRITERIAL THRESHOLD BINARIZATION OF CLUSTERED MATRICES AS EXEMPLIFIED BY EXPORT SECTOR'S COMPETITIVENESS OF THE SUBSAHARAN AFRICAN ECONOMIES

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Abstract: The paper offers the methodology of the matrix clustering consisting of multicriteria threshold binarization of the initial matrix of states of objects and clustering the resulting binary matrix into submatrices with different densities of zero and unit elements. Using hand calculation, the methodology was fine-tuned on the export competitiveness indicators of all the Sub-Saharan African countries for the Fresh Food sector of the Trade Competitiveness Map database. A standard R program was developed to implement this methodology and tested for all 14 export sectors of Sub-Saharan Africa, using the data from the Trade Competitiveness Map database for two sets of criteria. It was proposed to automate the procedure of fixing threshold criteria by using the K-Means clustering algorithm for two clusters consisting of zeros and ones.

Keywords: matrix clustering, binary matrix, multicriteria threshold binarization, Trade Competitiveness Map, Sub-Saharan African countries

I. INTRODUCTION

There are a large number of publications describing matrix clustering which consists in isolating a dense submatrix from a large sparse binary matrix whose elements consist of zeros and ones. With that, the dense submatrix mainly consists of ones. Such tasks arise in Data Mining, Web-analysis and image analysis of [1-4], analysis of bibliographic information flows [5], industrial design [6], gene analysis [7,8] and in other areas. To solve such problems, a ping-pong algorithm was proposed in [3], which means the optimal permutation of rows and columns in the original sparse binary matrix. Such kinds of tasks are reviewed in [9, 10].

An example of such a task in spatial economic analysis can be a task of constructing a symmetric matrix of mutual trade in a group of countries [11, 12] with its further binarization and matrix clusterization. Suppose we have a symmetric matrix of mutual exports (E_{ij}), where E_{ij} – exports from i – country to j – country, E_{ii} = 0, then this matrix can be transformed into a binary one $(\mathbf{B}_{ij})$ according to the rule:

$$B_{ij} = \begin{cases} 0, \text{if there are no exports from } i - \text{country to } j - \text{country}, \\ 1, \text{if there are exports from } i - \text{country to } j - \text{country}. \end{cases}$$

Afterwards, a dense submatrix can be singled out, consisting mainly of ones and corresponding to a group of countries with intense mutual trade. This is the way the matrix clustering is explained in [1-8].

At the same time, a class of problems arises in which the state matrices are constructed using heterogeneous data, described by different indicators.

Then the binarization of such matrices can be performed using criteria superimposed on a set of the values of all indicators. For example, [13] looks at the approach, the essence of which is that the set of indicators is grouped into a rectangular matrix, containing the numerical value of the studied indicator at the intersection of columns and rows. On closer inspection, it turns out that the researchers are forced to work with three-dimensional matrices. It is proposed to present the third dimension of the "intensity" of the indicator as a color gradient. Then the task of clustering is reduced to recognition of bit images. Simple neural networks easily cope with this task.

In this paper, this problem is referred to as a multicriteria threshold binarization of the state matrix. Clustering the obtained binary matrix is proposed to be performed according to the variation intervals in the number of zero or one elements of this matrix. Below, the methodology for clustering state matrices using multicriteria threshold binarization is described in [14].

Methodology of clustering state matrices using multicriteria threshold binarization

Such a methodology consists of three stages:

1. Construction of the state matrix

$$A = (A_{ij}) = \begin{vmatrix} A_{11} & A_{12} & A_{13} & \cdots & A_{1j} & \cdots & A_{1n} \\ A_{21} & A_{22} & A_{23} & \cdots & A_{2j} & \cdots & A_{2n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ A_{i1} & A_{i2} & A_{i3} & \cdots & A_{ij} & \cdots & A_{in} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ A_{m1} & A_{m2} & A_{m3} & \cdots & A_{mj} & \cdots & A_{mn} \end{vmatrix},$$

(1)

where A_{ij} – the value of an *j*-indicator for an *i*-object, m – the number of objects, *n* – the number of indicators, $1 \le i \le m, 1 \le j \le n$.

2. Multicriteria threshold binarization of the state matrix.

Suppose that all the indicators of the state matrix (1) are superimposed with some threshold criteria (K_j), which make it possible to convert this matrix into a binary matrix (B_{ij}).

Such a transformation will have the form

$$\begin{cases} (A_{ij}) \sim (B_{ij}), \\ B_{ij} = \begin{cases} 0, if\ A_{ij} \le K_j\ or\ A_{ij} \ge K_j & > 0 \\ 1, if\ A_{ij} > K_j\ or\ A_{ij} < K_j & > 0 \end{cases} \end{cases}$$

(2)

Here, with B_{ij} equaling zero, a less-than-or-equal-to sign is used, if A_{ij} is a stimulator, and a greater-than sign, if A_{ij} is a destimulator. With B_{ij} equaling one, the opposite signs are used. These criteria are introduced in order to abstract from insignificant values of the indicators.

3. Binary matrix clustering.

The clustering of the binary matrix is proposed to be conducted in the following way (Table 1):

Table 1: Binary Matrix Clustering (B_{ij}) by the Number of Zeros in Its Lines

Cluster 1 (Q_1 – first quartile)	from 0 to 25% zeros
Cluster 2 (Q_2 – second quartile)	from 25 to 50% zeros
Cluster 3 (Q_3 – third quartile)	from 50 to 75% zeros
Cluster 4 (Q_4 – fourth quartile)	from 75 to 100% zeros

If the distribution of the number of zeros is considered over twenty percentage intervals, then the binary matrix will be divided into five clusters, or binary submatrices, which differ by the density of zeros.

II. EMPIRICAL DATA

As an empirical basis for multi-criteria threshold binarization of the state matrix and further clustering, the WTO Trade Competitiveness Map for Sub-Saharan African countries will be used. On its basis, it is possible to construct state matrices for 14 export sectors of the economy with the dimension $m \times n$, where m is the number of countries in Sub-Saharan Africa, and n is the number of indicators. An example of such a state matrix with the dimension 45×19 for the Fresh food sector is shown in Table 2.

Table 2: Fresh Food Sector of Sub-Saharan African Countries Presented in the Form of State Matrix (A_{ij}) for 2016

Sector/ indicators	Fresh Food - 2016																		
	G_1	G_2	G_3	G_4	G_5	G_6	P_1	P_2	P_3	P_{4a}	P_{4b}	P_{5a}	P_{5b}	C_1	C_{1a}	C_{1b}	C_{1c}	C_{1d}	C_2
Country	Value	Value	Value	Value	Value	Value	Value	Value	Value	Value	Rank	Value	Rank	Value	Value	Value	Value	Value	Rank
Angola	42.328	-1%	0%	6%	-87%	1.4	-620.510	1.5	0.01%	4	118	5	125	0.0800%	-5.1600%	0.7700%	-14.1900%	18.6700%	26
Benin	245.029	1%	59%	40%	-62%	0.9	-826.527	22.5	0.03%	2	163	8	78	2.9300%	10.8100%	2.3400%	-6.6600%	-3.5700%	90
Botswana	114.647	6%	1%	3%	-33%	1.4	-113.481	50.9	0.02%	2	151	4	142	7.4900%	20.1300%	0.2300%	-2.3800%	-10.4900%	159
Burkina Faso	720.421	11%	28%	5%	57%	0.8	528.273	38.6	0.10%	3	145	6	107	12.7500%	19.2000%	-0.0400%	-0.2200%	-6.1900%	9
Burundi	61.396	-8%	49%	7%	11%	0.5	13.136	5.8	0.01%	2	168	5	129	-4.9000%	4.4600%	-10.2600%	-8.9300%	9.8300%	63
Cabo Verde	23.328	-17%	37%	10%	-43%	1.5	-35.215	43.2	0.00%	3	177	1	177	-10.3200%	-13.9900%	0.3400%	5.5200%	-2.1800%	10
Cameroon	1.096.142	4%	51%	14%	20%	0.9	379.396	46.8	0.15%	2	150	5	117	5.3100%	9.6300%	0.8300%	-0.8000%	-4.3500%	38
Chad	79.734	-3%	5%	3%	55%	1.1	56.798	5.5	0.01%	3	142	6	102	-1.4900%	5.2600%	1.0500%	-7.6700%	-0.1300%	3
Comoros	49.967	48%	71%	11%	5%	1.5	4.980	62.8	0.01%	2	174	4	130	82.4900%	50.2500%	1.0700%	31.6600%	-0.5000%	162
Congo, Dem. Rep.	53.741	0%	1%	7%	-70%	1.4	-256.417	0.7	0.01%	4	117	11	48	0.8400%	0.7900%	0.7400%	-1.7700%	1.0800%	12
Cote d'Ivoire	7.681.311	10%	74%	15%	72%	0.7	6.432.252	324.2	1.05%	3	130	12	41	11.8600%	13.3300%	1.4900%	0.8400%	-3.8000%	148
Djibouti	39.219	-8%	38%	6%	-74%	1.4	-234.401	41.6	0.01%	6	94	5	124	-5.0900%	-9.0400%	-0.0200%	-1.1800%	5.1500%	35
Equatorial Guinea	3.175	4%	0%	6%	-90%	0.9	-63.824	2.6	0.00%	3	158	2	162	4.8700%	-6.9500%	-1.2900%	-0.8100%	13.9200%	175
Eritrea	6.072	0%	2%	4%	-33%	1.4	-6.015	1.3	0.00%	4	170	2	173	1.1500%	6.0300%	-2.9500%	4.9900%	-6.9100%	6
Ethiopia	2.214.770	-1%	84%	6%	33%	0.8	1.110.966	21.6	0.30%	6	89	15	18	0.1000%	-1.2000%	0.5400%	1.6600%	-0.9000%	146
Gabon	20.875	-23%	0%	11%	-83%	0.9	-205.048	10.5	0.00%	3	152	8	98	-12.6400%	7.4700%	0.9300%	-8.3400%	-12.7000%	118
Gambia	19.992	23%	21%	16%	-51%	0.7	-42.940	9.8	0.00%	5	107	3	149	28.8300%	44.2000%	3.2500%	12.9000%	-31.5200%	61
Ghana	3.021.994	4%	28%	8%	50%	1.0	2.037.344	107.1	0.41%	2	159	10	54	5.6500%	2.8700%	0.6700%	3.0700%	-0.9600%	107
Guinea	219.664	-8%	10%	10%	-23%	1.4	-133.343	17.7	0.03%	8	76	9	77	-5.2600%	-3.8400%	0.2900%	-17.5500%	15.8400%	15
Guinea-Bissau	265.832	12%	97%	10%	76%	0.8	229.709	146.4	0.04%	2	167	2	165	14.3100%	-1.3100%	9.7200%	9.3600%	-3.4600%	172
Kenya	2.705.891	1%	56%	6%	47%	2.0	1.732.300	55.8	0.37%	5	100	11	49	2.1700%	0.1000%	-0.0300%	-0.5600%	2.6600%	115

Continuation Table 2

Lesotho	55.747	8%	6%	9%	-40%	0.9	-76.537	25.3	0.01%	6	103	2	159	9.6100%	14.7000%	0.4100%	-10.8100%	5.3200%	64
Liberia	239.067	-6%	25%	1%	20%	1.1	79.757	51.8	0.03%	2	149	5	127	-3.7700%	6.2000%	1.2800%	-4.8300%	-6.4200%	163
Mada-gascar	811.057	25%	35%	6%	62%	1.0	624.509	32.6	0.11%	4	124	7	88	31.8100%	5.2200%	2.6600%	9.9900%	13.9300%	140
Malawi	737.905	-5%	84%	6%	81%	1.1	661.389	40.8	0.10%	2	160	12	37	-2.8300%	-2.9700%	0.0700%	-2.6800%	2.7600%	138
Mali	289.929	-13%	26%	6%	21%	0.8	100.887	16.1	0.04%	3	141	5	110	-8.3500%	-9.9300%	-0.0800%	-17.7300%	19.3900%	128
Mauri-tania	607.716	1%	35%	8%	52%	1.1	416.749	141.3	0.08%	8	153	8	82	2.6900%	-1.2700%	-1.7300%	-1.5600%	7.2600%	111
Mauritius	231.685	17%	10%	14%	-47%	1.8	-424.134	183.4	0.03%	6	90	10	66	21.0400%	9.7400%	0.1100%	7.1500%	4.0400%	156
Mozam-bique	433.613	1%	12%	8%	-4%	0.0	-40.905	15.0	0.06%	4	127	19	8	2.9200%	7.8800%	1.4300%	-0.1200%	-6.2600%	129
Namibia	780.503	-6%	16%	4%	47%	0.8	504.500	314.8	0.11%	9	61	7	95	-3.6600%	-4.1100%	-0.2800%	-1.8700%	2.6000%	143
Niger	160.462	-1%	17%	10%	-10%	0.2	-35.832	7.8	0.02%	2	162	2	157	-0.2200%	8.3400%	-3.9100%	11.4100%	-16.0600%	57
Nigeria	573.596	-56%	1%	6%	-58%	0.0	-1.587.052	3.1	0.08%	4	129	7	100	19.2300%	-18.7500%	-3.5500%	-10.6100%	13.6800%	167
Rwanda	179.585	-2%	28%	7%	12%	1.0	39.157	15.1	0.02%	5	109	4	131	-0.4600%	3.9400%	-3.4300%	-4.4200%	3.4500%	91
Sao Tome and Pricipe	9.063	15%	86%	8%	-14%	0.9	-3.112	45.3	0.00%	1	176	1	175	18.1700%	-16.5000%	0.6600%	5.4800%	28.5300%	177
Senegal	574.496	8%	21%	12%	-8%	0.8	-100.564	37.7	0.08%	16	35	15	21	8.9900%	4.2400%	1.1300%	5.6700%	-2.0500%	60
Seychelles	221.887	157%	39%	16%	28%	0.8	98.874	2.343.6	0.03%	3	148	5	113	919.9200%	947.5400%	1.8400%	7.7500%	-37.2100%	17
Sierra Leone	171.550	21%	36%	14%	10%	1.2	32.356	23.2	0.02%	3	155	2	161	26.8600%	-1.6500%	0.2400%	4.1400%	24.1200%	44
Somalia	535.640	7%	94%	22%	9%	0.8	96.574	37.4	0.07%	4	116	2	152	7.8500%	-0.0500%	2.6900%	2.8400%	2.3700%	11
South Africa	5.143.152	1%	6%	4%	19%	1.5	1.669.567	92.0	0.70%	24	17	22	4	2.2600%	2.9000%	-0.5800%	3.3400%	-3.3900%	33
Swaziland	26.541	2%	1%	7%	-61%	1.1	-84.469	19.8	0.00%	8	73	6	108	3.7400%	6.7300%	0.6900%	-1.1400%	-2.5400%	100
Sudan	1.131.929	-6%	35%	9%	24%	1.0	443.612	21.8	0.16%	5	99	4	132	-3.5400%	-3.2200%	-2.5400%	-3.6500%	5.8700%	126
Togo	81.604	-8%	11%	5%	-4%	0.4	-8.080	10.7	0.01%	2	156	11	64	-5.1800%	-3.3300%	2.6700%	-4.0700%	-0.4500%	13
Uganda	1.002.562	4%	40%	4%	62%	0.8	773.795	24.2	0.14%	6	83	12	39	4.8500%	3.7500%	-0.4000%	-1.1300%	2.6300%	105
Zambia	398.905	-15%	7%	4%	41%	1.0	232.698	24.0	0.05%	4	114	8	80	-9.1600%	-3.5600%	-1.9700%	2.6500%	-6.2900%	56
Zimbabwe	995.859	-1%	35%	11%	23%	0.9	374.312	61.7	0.14%	1	173	1	172	-0.3700%	-1.8200%	0.7000%	-1.5500%	2.3100%	124

The dimension of this matrix is $m \times n = 45 \times 19$, where $m = 45$ is the number of countries in Sub-Saharan Africa that had statistical data for all 19 indicators. It should be noted that the Central African Republic, the Republic of Congo and Tanzania did not have such data for the sector of economy under study. In the paper, no detailed explanation of the 19 indicators will be provided, as this information is easily available on the ITC (International Trade Center, WTO) website, but only their original names will be used instead (Table 3).

Table 3: Descriptions of 19 Indicators

Current performance	General profile	Decomposition of changes in world market share (last 5 years)
P1. Value of net exports (in thousand US$)	G1. Value of exports (in thousand US$)	C1. Relative change of world market share, decomposed into: • C1a. Competitiveness effect (%); • C1b. Initial geographic specialization (%); • C1c. Initial product specialization (%); • C1d. Adaptation effect (%)
P2. Per capita exports (US$/inhabitant)	G2. Trend growth of exports (last 5 years) (%)	
P3. Share in world market (% share of world exports)	G3. Share in national exports (%)	
P4.a. Product diversification (N° of equivalent products)	G4. Share in national imports (%)	
P4.b. Product concentration (Spread)	G5. Growth in per capita exports (last 5 years) (%)	
P5.a. Market diversification (N° of equivalent markets)	G6. Level in relative unit values (world average = 1)	C2. Matching with dynamics of world demand
P5.b. Market concentration (Spread)		

Let's formulate hypothetic criteria derived from heuristic considerations (the selection of the criteria can be different).

If $G_1 \leq 30{,}000$, $G_2 \leq 2\%$, $G_3 \leq 10\%$, $G_4 \geq 10\%$, $G_5 \leq 10\%$, $G_6 \leq 1$, $P_1 \leq 50$, $P_2 \leq 10$, $P_3 \leq 0.05\%$; $P_{4a} \leq 2$, $P_{4b} \geq 100$, $P_{5a} \leq 5$, $P_{5b} \geq 100$, $C_1 \leq 5\%$, $C_{1a} \leq 5\%$, $C_{1b} \leq 2\%$, $C_{1c} \leq 5\%$, $C_{1d} \leq 5\%$, $C_2 \geq 100$, then $B_{ij} = 0$, otherwise $B_{ij} = 1$.

Here G_1, P_1 are taken as absolute values (thousands of US dollars), G_6, P_{4a}, P_{4b}, P_{5a}, P_{5b}, C_2 – as relative non-interest units, P_2 – as a ratio (exports per capita), G_2, G_3, G_4, G_5, P_3, C_{1a}, C_{1b}, C_{1c}, C_{1d} – as relative percentage units, P_{4b}, P_{5b}, C_2 –as ranks (positions in the ranking of all the world's countries in the sector under review according to the values of these indicators).

The rank indicators P_{4b}, P_{5b}, C_2 and the import indicator G_4 (share of the sector in question in the national imports) were considered as destimulators.

Applying these criteria to the initial state matrix (Table 2), we obtain a binary matrix (Table 4).

Table 4: Binary Matrix Built by Applying Hypothetic Criteria to the Initial State Matrix (Table 2)

Sector/indicator	Fresh Food - 2016																		
Countries	G1	G2	G3	G4	G5	G6	P1	P2	P3	P4a	P4b	P5a	P5b	C1	C1a	C1b	C1c	C1d	C2
	Value	Value	Value	Value	Value	Value	Value	Value	Value	Value	Rank	Value	Rank	Value	Value	Value	Value	Value	Rank
Angola	1	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0	0	1	1
Benin	1	0	1	0	0	0	0	1	0	0	0	1	1	0	1	1	0	0	1
Botswana	1	1	0	1	0	1	0	1	0	0	0	0	0	1	1	0	0	0	0
Burkina Faso	1	1	1	1	1	0	1	1	1	1	0	1	0	1	1	0	0	0	1
Burundi	1	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	1	1
Cabo-Verde	0	0	1	0	0	1	0	1	0	1	0	0	0	0	0	0	1	0	1
Cameroon	1	1	1	0	1	0	1	1	1	0	0	0	0	1	1	0	0	0	1
Chad	1	0	0	1	1	1	1	0	0	1	0	1	0	0	1	0	0	0	1
Comoros	1	1	1	0	0	1	1	1	0	0	0	0	0	1	1	0	1	0	0
Congo DR	1	0	0	1	0	1	0	0	0	1	0	1	1	0	0	0	0	0	1
Cote d'Ivoire	1	1	1	0	1	0	1	1	1	1	0	1	1	1	1	0	0	0	0
Djibouti	1	0	1	1	0	1	0	1	0	1	1	0	0	0	0	0	0	1	1
Equatorial Guinea	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0
Eritrea	0	0	0	1	0	1	0	0	0	1	0	0	0	0	1	0	0	0	1
Ethiopia	1	0	1	1	1	0	1	1	1	1	1	1	1	0	0	0	0	0	0
Gabon	0	0	0	0	0	0	0	1	0	1	0	1	1	0	1	0	0	0	0
Gambia	0	1	1	0	0	0	0	0	0	1	0	0	0	1	1	1	1	0	1
Ghana	1	1	1	1	1	0	1	1	1	0	0	1	1	1	0	0	0	0	0
Guinea	1	0	0	0	0	1	0	1	0	1	1	1	1	0	0	0	0	1	1
Guinea-Bissau	1	1	1	0	1	0	1	1	0	0	0	0	0	1	0	1	1	0	0
Kenya	1	0	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0
Lesotho	1	1	0	1	0	0	0	1	0	1	0	0	0	1	1	0	0	1	1
Liberia	1	0	1	1	1	1	1	1	0	0	0	0	0	0	1	0	0	0	0

Continuation Table 4

Madagascar	1	1	1	1	1	0	1	1	1	1	0	1	1	1	1	1	1	1	0
Malawi	1	0	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0	0
Mali	1	0	1	1	1	0	1	1	0	1	0	0	0	0	0	0	0	1	0
Mauritania	1	0	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	1	0
Mauritius	1	1	0	0	0	1	0	1	0	1	1	1	1	1	1	0	1	0	0
Mozambique	1	0	1	1	0	0	0	1	1	1	0	1	1	0	1	0	0	0	0
Namibia	1	0	1	1	1	0	1	1	1	1	1	1	1	0	0	0	0	0	0
Niger	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1
Nigeria	1	0	0	1	0	0	0	0	1	1	0	1	0	1	0	0	0	1	0
Rwanda	1	0	1	1	1	0	1	1	0	1	0	0	0	0	0	0	0	0	1
Sao-Tome and Pricipe	0	1	1	1	0	0	0	1	0	0	0	0	0	1	0	0	1	1	0
Senegal	1	1	1	0	0	0	0	1	1	1	1	1	1	1	0	0	1	0	1
Seychelles	1	1	1	0	1	0	1	1	0	1	0	0	0	1	1	0	1	0	1
Sierra Leone	1	1	1	0	0	1	1	1	0	1	0	0	0	1	0	0	0	1	1
Somalia	1	1	1	0	0	0	1	1	1	1	0	0	0	1	0	1	0	0	1
South Africa	1	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1
Swaziland	0	0	0	1	0	1	0	1	0	1	1	1	0	0	1	0	0	0	0
Sudan	1	0	1	1	1	0	1	1	1	1	1	0	0	0	0	0	0	1	0
Togo	1	0	1	1	0	0	0	1	0	0	0	1	1	0	0	1	0	0	1
Uganda	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	0	0	0	0
Zambia	1	0	0	1	1	0	1	1	0	1	0	1	1	0	0	0	0	0	1
Zimbabwe	1	0	1	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0

Clustering this matrix into four quartets resulted in the following four clusters (Table 5).

Table 5. Binary Matrix Clustering (Table 4)

Clusters	Sub-Saharan African Countries
Cluster 1	Madagascar
Cluster 2	Burkina Faso, Cameroon, Cote d'Ivoire, Ethiopia, Ghana, Kenya, Malawi, Mauritania, Mauritius, Namibia, Senegal, Seychelles, Sierra Leone, Somalia, South Africa, Sudan, Uganda
Cluster 3	Angola, Benin, Botswana, Burundi, Cabo Verde, Chad, Comoros, Congo, Dem. Rep., Djibouti, Eritrea, Gabon, Gambia, Guinea, Guinea-Bissau, Lesotho, Liberia, Mali, Mozambique, Niger, Nigeira, Rwanda, Sao Tome and Pricipe, Swaziland, Togo, Zambia, Zimbabwe
Cluster 4	Equatorial Guinea

As can be seen from Table 5, most countries of Sub-Saharan Africa are concentrated in the second and third clusters. It should be noted that the selected clusters show the scaled-up competitiveness of the Fresh Food export sector of Sub-Saharan Africa, which increases from the fourth cluster to the first one.

Let's tighten the criteria by changing their values by a factor of 2, except for G_6: $G_1 \leq 60{,}000$, $G_2 \leq 4\%$, $G_3 \leq 20\%$, $G_4 \geq 5\%$, $G_5 \leq 20\%$, $G_6 \leq 1$, $P_1 \leq 100$, $P_2 \leq 20$, $P_3 \leq 0.1\%$; $P_{4a} \leq 4$, $P_{4b} \geq 50$, $P_{5a} \leq 10$, $P_{5b} \geq 50$, $C_1 \leq 10\%$, $C_{1a} \leq 10\%$, $C_{1b} \leq 4\%$, $C_{1c} \leq 10\%$, $C_{1d} \leq 10\%$, $C_2 \geq 50$, then there will be a new clustering of Sub-Saharan African countries in the sector under review (Table 6).

Table 6. Binary Matrix Clustering of Fresh Food Sector with Criteria Changed

Clusters	Sub-Saharan African Countries
Cluster 1	
Cluster 2	Cote d'Ivoire, Kenya, Uganda, South Africa,
Cluster 3	Botswana, Burkina Faso, Cameroon, Chad, Comoros, Djibouti, Ethiopia, Gambia, Ghana, Guinea, Guinea Bissau, Liberia, Madagascar, Malawi, Mali, Mauritania, Mauritius, Sao Tome and Pricipe, Senegal, Seychelles, Sierra Leone, Somalia, Namibia, Zambia, Zimbabwe, Sudan
Cluster 4	Angola, Benin, Burundi, Cabo Verde, Congo Dem.Rep., Equatorial Guinea, Eritrea, Gabon, Lesotho, Mozambique, Niger, Nigeria, Rwanda, Swaziland, Togo

As one would expect, most of the countries, in comparison with the previous clustering (Table 5), moved to the less competitive clusters.

Below there will be a description of an R-program developed by the authors for multicriteria threshold binarization of the state matrix and further clustering of the binary matrix, fine-tuned on the basis of the initial state matrix (Table 2), using the first set of criteria.

Development of multicriteria threshold binarization and clustering of matrices

To create a binary matrix using the R language (R version 3.4.4) [15], BinMat function was written, its name coming from "binary matrix". Below is the code for this function (Figure 1).

```
BinMat <- function(Mat, Thres, Dest = rep(F, ncol(Mat))) {
    Mat <- rbind(Dest, Thres, Mat)
    Mat <- apply(as.matrix(Mat), 2, function(x) {
        if (x[1] == 1) {
            ifelse(x >= x[2], 0, 1)
        } else {
            ifelse(x <= x[2], 0, 1)
        }
    })
    return(Mat[-c(1, 2), ])
}
```

Figure 1: Function code for matrix binarization

Function arguments

The function has three arguments, used to specify the initial data for binarization, the binarization thresholds for each variable, and to indicate which variables are destimulators, and which ones are stimulators.

`Mat` – an object of the data table class to be binarized, the name deriving from the word "matrix". Instead of the data table, one can use a matrix class object. If a data table is used, then all its variables must be numeric. For matrices we will further use the following vectors.

`Thres` – a vector of numeric values that assigns the binarization thresholds for all the variables in the data table (columns of the matrix). The name of the argument derives from the word "threshold". The length of the vector must match the number of variables in the data table or columns in the matrix.

`Dest` – a vector of logical values indicating variable destimulators and variable stimulators. The name of the argument derives from the word "destimulator". The length of the vector must match the number of variables in the data table. The variable destimulator is marked as TRUE (an abbreviated T can be used), the variable stimulator is marked as FALSE (an abbreviated F can be used).

Instead of the vector of logical values, a vector of integer values can be used. Then the destimulators are denoted by the number 1, and the stimulators – by the number 0.

By default, all variables are stimulators. In this case, the Dest argument can be left blank.

Code operation description

Binarization of the matrix is carried out in three operations.

The first operation is a line-by-line connection of the `Dest` and `Thres` vectors to the `Mat` data table. This is done by means of the `rbind` function, for which the lines to be joined are listed in the order from the first to the last: `Dest`, `Thres`, `Mat`.

Using the `rbind` function results in overwriting the original `Mat` data table. The `Dest` vector becomes the first observation in the overwritten data table, and the `Thres` vector becomes the second observation. Then come the same observations in the same order as they were in the original `Mat` object. When the vector `Dest` is added to the data table, the `T` values in the former are replaced by 1, and the F values are replaced by 0.

The second stage is binarization itself, with is carried out by using the `apply` function, which makes it possible to apply the same operation to each column or row. In our case, it is applied to the columns, for this, the value of the second argument of the `apply` function is 2. The first argument for `apply` is the original data table. It is converted into the matrix right on entry, using the `as.matrix` function. This is done to speed up the processing of large data. In R, matrices are processed faster than the data tables.

The binarization procedure is described by the third argument of the `apply` function. In our case, there is an anonymous function there, which uses a pair of `ifelse` functions to specify a condition and two variant actions that are applied when the condition is met, and when it is not.

The given condition is whether the parameter is a destimulator, that is, whether the first value in the column is 1. If this is the case, then by using the `ifelse` function, each value in the column is compared with the second value (the binarization threshold). All values that exceed or equal the binarization threshold are replaced by 0. All values that are less than the binarization threshold are replaced by 1 (Formula 2).

If the indicator is a stimulator (the first value in the column is 0), then by using the `ifelse` function, each value in the column is also compared with the second value (the binarization threshold). All values that are less than or equal to the binarization threshold are replaced by 0. All values that are greater than the binarization threshold are replaced by 1 (Formula 2).

At the third stage of the function the authors created, it returns the result of binarization in the form of a matrix. In the process of returning the result, the first two lines, added at the first stage of processing the data, are cut off from the result.

Below is the code showing how the created function can be used.

```
BinResult <- BinMat(Mat = MyData,
Thres = MyThres, Dest = MyDest)
```

The result of using the function, displayed on the console desk, is shown in Figure 2.

```
> BinMat(Mat=Pril1,Thres=Thres,Dest=TYP)
                  G1 G2 G3 G4 G5 G6 P1 P2 P3 P4a P4b P5a P5b C1 C1a C1b C1c C1d C2
Angola             1  0  0  0  0  1  0  0  0   0   0   0   0  0   0   0   0   1  1
Benin              1  0  1  0  0  0  0  1  0   0   0   0   0  0   1   0   0   0  0
Botswana           1  1  0  1  0  1  0  1  0   0   0   0   0  0   1   0   0   0  0
Burkina_Faso       1  1  1  0  1  0  1  1  0   0   0   0   0  1   1   0   0   0  1
Burundi            1  0  1  0  0  0  1  0  0   0   0   0   0  0   0   0   0   0  0
Cabo_Verde         0  0  1  0  0  1  0  1  0   0   0   0   0  0   0   0   0   0  1
Cameroon           1  1  1  0  0  0  1  1  1   0   0   0   0  0   0   0   0   0  1
Chad               1  0  0  1  1  1  1  0  0   0   0   0   0  0   0   0   0   0  1
Comoros            1  1  1  0  0  1  1  1  0   0   0   0   0  1   1   0   1   0  0
Congo_Dem._Rep.    1  0  0  0  0  1  0  0  0   0   0   1   1  0   0   0   0   0  1
Cote_d'Ivoire      1  1  1  0  1  0  1  1  1   0   0   1   1  1   1   0   0   0  0
Djibouti           1  0  1  0  0  1  0  1  0   1   0   0   0  0   0   0   0   0  1
Equatorial_Guinea  0  1  0  0  0  0  0  0  0   0   0   0   0  0   0   0   0   1  0
Eritrea            0  0  0  1  0  1  0  0  0   0   0   0   0  0   0   0   0   0  1
Ethiopia           1  0  1  0  1  0  1  1  1   1   0   1   1  0   0   0   0   0  0
Gabon              0  0  0  0  0  0  0  0  0   0   0   0   0  0   0   0   0   0  0
Gambia             0  1  1  0  0  0  0  0  0   1   0   0   0  1   1   0   1   0  0
Ghana              1  1  1  0  1  0  1  1  1   0   0   0   0  0   0   0   0   0  0
Guinea             1  0  0  0  0  1  0  0  0   1   0   0   0  0   0   0   0   1  1
GuineaBissau       1  1  1  0  1  0  1  1  0   0   0   0   0  1   0   1   0   0  0
Kenya              1  0  1  0  1  1  1  1  1   1   0   1   1  0   0   0   0   0  0
Lesotho            1  1  0  0  0  0  0  1  0   1   0   0   0  0   1   0   0   0  0
Liberia            1  0  1  1  0  1  1  1  0   0   0   0   0  0   0   0   0   0  0
```

Figure 2: Result of using the function to binarize the matrix

The function code to separate binary matrix rows into classes

To distribute the rows of a binary matrix over a given number of classes in the R language, a function was written. The name of the function derives from the phrase "binary matrix" and the word "quantification". Below is the code for this function (Figure 3).

```
BMQuant <- function(BM, step = 25)
{
    M <- ceiling((apply(BM, 1, function(x) 100 -
        sum(x)/length(x) * 100))/step)
    return(matrix(M, nrow = nrow(BM), 1,
        dimnames = list(rownames(BM), c("Класс"))))
}
```

Figure 3: Function code to distribute binary matrix rows into classes

Function arguments

The function has two arguments, which are used to indicate the binary matrix and the gradation width of the number of zeros in a row of the binary matrix.

`BM` is an object of the matrix class; contains a binary matrix. The name of the argument derives from the phrase "binary matrix".

`step` is an integer, which must be more than zero. The argument can be omitted, in which case it is 25 by default. The argument gets its name from the word "step". The argument indicates the gradation width of the number of zeros. It is to be shown as a percentage.

Code operation description

The function works as follows. For each row of the binary matrix, the percentage of zeros is calculated. For this, the sum of the values in a row divided by the length of the row and multiplied by 100 is subtracted from 100. In the body of the `BMQuant` function, the following code fragment corresponds to this operation: `100 – sum(x)/length(x) * 100`

To carry out the above operation with each row, the `apply` function is used, and the above code fragment is used in it as an anonymous function:

```
apply(BM, 1, function(x) 100 –
    sum(x)/length(x) * 100)
```

Next, the calculated percentage of zeros is divided by the value specified by the `step` argument. The resulting value is rounded up to an integer by means of the `ceiling` function.

The obtained result is returned by the `BMQuant` function in the form of a matrix with one column and the number of rows which is equal to the number of rows in the original binary matrix. The row names are taken from the original binary matrix. Below is the code showing an example of using the created function.

```
Result <- BMquant2(BM = BM1, step =
                  25)
```

The result of using the function, displayed on the console desk, is shown in Figure 4

```
> BMquant2(BM=BM1,step=25)
                     Класс
Angola                   3
Benin                    3
Botswana                 3
Burkina_Faso             2
Burundi                  3
Cabo_Verde               3
Cameroon                 3
Chad                     3
Comoros                  3
Congo_Dem._Rep.          3
Cote_d'Ivoire            2
Djibouti                 3
Equatorial_Guinea        4
Eritrea                  3
```

Figure 4. Result of using the function code to separate rows of a binary matrix into classes

The developed program was used by the authors for multicriteria threshold binarization of state matrices and their clustering for all 14 export sectors of the economies of Sub-Saharan African countries.

III. RESULTS OF MATRIX CLUSTERING OF EXPORT SECTORS OF ECONOMIES OF SUB-SAHARAN AFRICA

The country clustering of the initial state matrices, carried out by means of the developed program of multicriteria threshold binarization according to the scale presented in Table 1, for all economic sectors of Sub-Saharan Africa and the first set of criteria, is shown in Table 7.

Table 7: Distribution of Sub-Saharan African Countries by Sector and Cluster (First Set of Criteria)

Sector	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Fresh food	Madagascar	Burkina Faso, Cameroon, Cote d'Ivoire, Ethiopia, Ghana, Kenya, Malawi, Mauritania, Mauritius, Namibia, Senegal, Seychelles, Sierra Leone, Somalia, South Africa, Sudan, Uganda	Angola, Benin, Botswana, Burundi, Cabo Verde, Chad, Comoros, Congo Dem. Rep., Djibouti, Eritrea, Gabon, Gambia, Guinea, Guinea-Bissau, Lesotho, Liberia, Mali, Mozambique, Niger, Nigeira, Rwanda, Sao Tome and Pricipe, Swaziland, Togo, Zambia, Zimbabwe	Equatorial Guinea
Processed food		Malawi, Mauritius, South Africa, Togo	Benin, Botswana, Burkina Faso, Burundi, Cabo Verde, Cote d'Ivoire, Cameroon, Comoros, Ethiopia, Gambia, Ghana, Guinea, Kenya, Mali, Mauritania, Mozambique, Namibia, Niger, Nigeria, Rwanda, Senegal, Seychelles, Somalia, Swaziland, Sudan, Uganda, Zambia	Congo Dem. Rep., Congo Rep., Djibouti, Gabon, Lesotho, Madagascar, Zimbabwe
Wood products		Cameroon, Congo Rep., Cote d'Ivoire, Equatorial Guinea, Gabon, South Africa	Botswana, Central African Republic, Congo Dem. Rep., Djibouti, Ghana, Kenya, Liberia, Madagascar Mauritius, Mozambique, Namibia, Nigeria, Senegal, Swaziland, Togo, Uganda, Zambia	Benin, Ethiopia, Lesotho, Malawi, Sudan, Zimbabwe
Textiles			Cameroon, Ethiopia, Ghana, Kenya, Madagascar, Mauritius, Niger, Nigeria, Senegal, South Africa, Togo, Uganda	Benin, Botswana, Burkina Faso, Cote d'Ivoire, Gambia, Lesotho, Mozambique, Namibia, Swaziland, Zambia, Zimbabwe

Chemicals			Benin, Botswana, Burkina_Faso, Equatorial Guinea, Ghana, Kenya, Madagascar, Mauritius, Mozambique, Niger, Nigeria, Rwanda, Senegal, South Africa, Swaziland, Togo, Uganda	Burundi, Cameroon, Congo Dem. Rep., Congo_Rep., Cote d'Ivoire, Djibouti, Ethiopia, Gabon, Malawi, Mali, Namibia, Somalia, Sudan, Zambia, Zimbabwe
Leather products			Cabo Verde, Cote d'Ivoire, Ethiopia, Ghana, Kenya, Lesotho, Madagascar, Mauritius, Namibia, Rwanda, Senegal, South Africa, Nigeria, Uganda, Zambia, Zimbabwe	Burundi, Mali, Somalia, Sudan
Basic manufactures		South Africa, Madagascar,	Benin, Botswana, Burkina Faso, Burundi, Cameroon, Congo Dem. Rep., Congo Rep., Ethiopia, Gabon, Ghana, Kenya, Mauritius, Mozambique, Namibia, Nigeria, Rwanda, Senegal, Swaziland, Togo, Uganda, Zambia, Zimbabwe	Cote d'Ivoire, Liberia, Malawi, Sudan
Non-electronic machinery		Swaziland	Benin, Botswana, Burkina Faso, Cameroon, Congo Rep., Cote d'Ivoire, Gabon, Guinea, Liberia, Mauritius, Mozambique, Namibia, Rwanda, South Africa, Togo, Uganda, Senegal, Madagascar, Zimbabwe	Congo Dem. Rep., Equatorial Guinea, Ghana, Kenya, Malawi, Mali, Niger, Nigeria, Sierra Leone, Sudan, Zambia,
IT and Consumer electronics		Rwanda, South Africa	Botswana, Cote d'Ivoire, Gabon, Kenya, Madagascar, Malawi, Mali, Mauritius, Namibia, Senegal, Zambia, Zimbabwe	Congo Dem. Rep., Ghana, Lesotho, Niger, Uganda
Electronic components		Mali, Swaziland	Botswana, Cameroon, Congo Rep., Cote d'Ivoire, Gabon, Kenya, Liberia, Madagascar, Mauritius, Namibia, South Africa, Uganda	Niger, Ghana, Lesotho, Malawi, Senegal, Zambia, Zimbabwe
Transport equipment		Benin, South Africa,	Botswana, Burkina Faso, Cameroon, Congo Rep., Cote d'Ivoire, Equatorial Guinea, Gabon, Ghana, Kenya, Liberia, Madagascar, Mali, Mauritius, Mozambique, Namibia, Niger, Rwanda, Senegal, Swaziland, Sudan, Togo, Uganda, Zimbabwe	Congo Dem. Rep., Guinea, Malawi, Seychelles, Sierra Leone, Zambia
Clothing	Madagascar	Lesotho, Mauritius, Swaziland, Uganda,	Cabo Verde, Cote d'Ivoire, Ethiopia, Ghana, Kenya, Namibia, Senegal, South Africa, Zimbabwe	Botswana, Eritrea, Malawi
Miscellaneous manufacturing		Senegal, Togo	Benin, Botswana, Burkina Faso, Cameroon, Congo Dem. Rep., Congo Rep., Cote d'Ivoire, Ethiopia, Ghana, Guinea, Kenya, Lesotho, Liberia, Madagascar, Mali, Mauritius, Namibia, Niger, Rwanda, Seychelles, Uganda, South Africa,	Gabon, Mozambique, Malawi, Nigeria, Sierra Leone, Swaziland, Sudan, Zambia, Zimbabwe
Minerals	Rwanda	Botswana, Congo Dem. Rep., Cote d'Ivoire, Equatorial Guinea, Eritrea, Gabon, Ghana, Guinea, Lesotho, Liberia, Mozambique, Namibia, Niger, South Africa, Senegal, Zambia	Angola, Benin, Burkina Faso, Burundi, Cameroon, Chad, Congo Rep., Ethiopia, Kenya, Madagascar, Malawi, Mauritania, Mauritius, Nigeria, Seychelles, Swaziland, Sudan, Togo, Uganda, Zimbabwe	Mali

As can be seen from this table, Madagascar got into the first most competitive cluster for Fresh Food and Clothing sectors, and Rwanda got there for the Minerals sector. It should be noted that the countries with no export sectors of the economy should be referred to the worst fourth cluster (with the values of all 19 indicators being zero). For example, in the Processed Food sector, 7 countries (Angola, Chad, Equatorial Guinea, Eritrea, Guinea-Bissau, Liberia, Sao Tome and Principe) are absent, but they are represented in the Fresh Food sector. So, these 7 countries can be referred to cluster 4 of the Processed Food sector. The same will hold true for the other sectors.

On the basis of Table 7, sectoral clusters can be built, showing the degree of competitiveness of the export sectors of the economy. For this, let's enter the indicator of the sectoral competitiveness by formula

$$I_{sec}=0.4\,N_1+0.3\,N_2+0.2\,N_3+0.1\,N_4, \quad (3)$$

where N_i – the number of countries falling into an i - country cluster, 0.4 – the weighting factor of the first country cluster, 0.3 – the weighting factor of the second country cluster, 0.2 – the weighting factor of the third country cluster, 0.1 – the weighting factor of the fourth country cluster.

In the formula (3), the weighting factors, depending on the country clusters, were taken with a uniform step (0.1), with their sum equaling to one.

Since the total number of countries in Sub-Saharan Africa was 45, then supposing that they all fall into cluster 1, the maximum value can be obtained I_{sec}=45x0.4=18. By dividing the interval $0\leq I_{sec}\leq 18$ into four equal intervals, let's introduce the following scale for assessing sectoral competitiveness (Table 8).

Table 8: Scale for Assessing Sectoral Competitiveness

Measurement interval I_{sec}	Number of sectoral clusters	Features of sectoral clusters
$0\leq I_{sec}\leq 4.5$	4	Low competitiveness
$4.5\leq I_{sec}\leq 9$	3	Competitiveness below the average
$9< I_{sec}\leq 13.5$	2	Competitiveness above the average
$13.5< I_{sec}\leq 18$	1	High competitiveness

When describing the characteristics of the sectoral clusters, the average level of competitiveness was assumed to be 18/2 = 9.

Now, basing on Tables 7 and 8, let's construct the breakdown of the number of countries in Sub-Saharan Africa by sector and country cluster, simultaneously identifying sectoral clusters (Table 9)

Table 9. Distribution of Number of Sub-Saharan African Countries by Sector and Cluster (First Set of Criteria)

Sector	Country clusters				Countries in total	I_{sec}	Sectoral clusters
	Cluster 1	Cluster 2	Cluster 3	Cluster 4			
Fresh food	1	17	26	1	45	10.8	2
Processed food		4	27	7	38	7.3	3
Wood products		6	17	6	29	5.8	3
Textiles			12	11	23	3.5	4
Chemicals			17	15	32	4.9	3
Leather products			16	4	20	3.6	4
Basic manufactures		2	22	4	28	5.4	3
Non-electronic machinery		1	19	11	31	5.2	3
IT and Consumer electronics		2	12	5	19	3.5	4
Electronic components		2	12	7	21	3.7	4
Transport equipment		2	23	6	31	5.8	3
Clothing	1	4	9	3	17	3.7	4
Miscellaneous manufacturing		2	22	9	33	5.9	3
Minerals	1	16	20	1	38	9.3	2

As can be seen from Table 9, only Fresh food and Minerals sectors, which are the most developed for Sub-Saharan Africa, fell into the second sectoral cluster.

Table 10 lists the best and worst positions of Sub-Saharan African countries in the export sectors of the economy. For this table, three best and three worst positions were taken from the corresponding sectoral binary matrices. The sum of 1-elements in the rows of these matrices is shown in brackets. South Africa had most of the superior positions in the export sectors of the economy (11 out of 14), whereas Malawi had most of the inferior ones (9 out of 14).

Table 10: Superior and Inferior Positions of Sub-Saharan African Countries in Export Sectors of Economy (First Set of Criteria)

Sector	Superior positions	Inferior positions
Fresh food	Madagascar (16) Burkina Faso (13), Cote d'Ivoire (12), Mauritania (12), Senegal (12), South Africa (12), Uganda (12),	Angola (6), Cabo Verde (6), Zimbabwe (6), Eritrea (5), Gabon (5), Niger (5), Equatorial Guinea (4)
Processed food	Malawi (13), South Africa (11), Ethiopia (10), Mauritius (10), Togo (10)	Botswana (5), Cabo Verde (5), Somalia (5), Congo Dem. Rep. (4), Gabon (4), Lesotho (4), Madagascar (4), Zimbabwe (4), Djibouti (3), Congo Rep. (3)
Wood products	Congo Rep. (13), Equatorial Guinea (12), South Africa (12), Cameroon (11)	Malawi (4), Sudan (4), Benin (3), Ethiopia (3), Lesotho (2),
Textiles	Niger (9), South Africa (8), Mauritius (8), Ghana (7),	Zambia (4), Zimbabwe (4), Mozambique (4), Burkina Faso (3), Cote d'Ivoire (3), Lesotho (3), Namibia (3), Benin (2), Botswana (2), Gambia (2),
Chemicals	Uganda (9), Mauritius (9), South Africa, Togo (8), Mozambique (8), Ghana (8), Rwanda (7), Niger (7), Equatorial Guinea (7), Benin (7)	Burundi (3), Congo Rep. (3), Zambia (3), Malawi (2), Mali (2), Somalia (1), Sudan (1)
Leather products	Mauritius (9), South Africa (8), Kenya (8), Senegal (7), Uganda (7), Rwanda (7), Ethiopia (7), Cote d'Ivoire (7)	Namibia (5), Nigeria (5), Cabo Verde (5), Madagascar (5), Mali (4), Burundi (4), Sudan (4), Somalia (2)
Basic manufactures	South Africa (13), Madagascar (13), Botswana (10), Congo Rep. (9), Gabon (9), Zambia (9)	Togo (5), Kenya (5), Rwanda (5), Malawi (4), Cote d'Ivoire (4), Sudan (3), Liberia (3)
Non-electronic machinery	Swaziland (12), Togo (9), Rwanda (9), Liberia (8), Uganda (8), South Africa (8), Benin (8)	Malawi (3), Sierra Leone (3), Sudan (3), Congo Dem. Rep. (3), Equatorial Guinea (2), Mali (2), Nigeria (1),
IT and Consumer electronics	Rwanda (11), South Africa (10), Malawi (9), Mali (9), Mauritius (9)	Madagascar (6), Namibia (6), Zambia (6), Congo Dem. Rep. (4), Ghana (4), Lesotho (3), Niger (3), Uganda (3),
Electronic components	Swaziland (11), Mali (10), Liberia (9), Madagascar (9),	Congo Rep. (5), Lesotho (5), Mauritius (5), Senegal (4), Malawi (4), Zimbabwe (4), Ghana (3), Niger (3), Zambia (3),
Transport equipment	South Africa (12), Benin (10), Uganda (9)	Guinea (5), Liberia (5), Sudan (5), Mali (5), Zimbabwe (5), Zambia (4), Malawi (4), Sierra Leone (4), Congo Dem. Rep. (2), Seychelles (2)
Clothing	Madagascar (15), Mauritius (13), Lesotho (12), Uganda (12)	Botswana (4), Malawi (3), Eritrea (2)
Miscellaneous manufacturing	Togo (11), Senegal (10), Burkina Faso (9)	Nigeria (4), Sierra Leone (4), Gabon (4), Zambia (4), Sudan (3), Zimbabwe (3), Malawi (2),
Minerals	Rwanda (15), Botswana (13), Namibia (13), Niger (13), South Africa (13), Cote d'Ivoire (12), Equatorial Guinea (12), Lesotho (12), Mozambique (12)	Benin (6), Burundi (6), Cameroon (6), Togo (6), Malawi (6), Uganda (6), Ethiopia (5), Seychelles (5), Mali (4)

The similar calculations for the second set of criteria with breakdown by sectors, clusters, superior and inferior positions are given in Tables 11-13.

Table 11: Breakdown of Sub-Saharan African Economies by Sector and Cluster (Second Set of Criteria)

Sector	Cluster 2	Cluster 3	Cluster 4

Fresh food	Cote d'Ivoire, Kenya, Uganda, South Africa	Botswana, Burkina Faso, Cameroon, Chad, Comoros, Djibouti, Ethiopia, Gambia, Ghana, Guinea, GuineaBissau, Liberia, Madagascar, Malawi, Mali, Mauritania, Mauritius, Namibia, Sao Tome and Pricipe, Senegal, Seychelles, Sierra Leone, Somalia, Sudan, Zambia, Zimbabwe	Angola, Benin, Burundi, Cabo Verde, Congo Dem. Rep., Equatorial Guinea, Eritrea, Gabon, Lesotho, Mozambique, Niger, Nigeria, Rwanda, Swaziland, Togo
Processed food	South Africa	Comoros, Rwanda, Senegal, Seychelles, Togo, Swaziland, Niger, Mauritania, Mauritius, Malawi, Ethiopia, Gambia, Kenya	Benin, Botswana, Burkina Faso, Burundi, Cabo Verde, Cameroon, Congo Dem. Rep., Congo Rep., Cote d'Ivoire, Djibouti, Ghana, Guinea, Gabon, Lesotho, Mozambique, Mali, Madagascar, Nigeria, Namibia, Somalia, Sudan, Uganda, Zambia, Zimbabwe
Wood products	South Africa	Cameroon, Central African Republic, Congo Rep., Congo Dem. Rep., Cote d'Ivoire, Djibouti, Equatorial Guinea, Gabon, Ghana, Liberia, Mauritius, Swaziland, Uganda, Zambia	Benin, Botswana, Ethiopia, Kenya, Lesotho, Madagascar, Malawi, Mozambique, Namibia, Nigeria, Senegal, Sudan, Togo, Zimbabwe
Textiles		Madagascar, Mauritius, Niger, South Africa	Benin, Botswana, Burkina Faso, Cameroon, Cote d'Ivoire, Ethiopia, Gambia, Ghana, Kenya, Lesotho, Mozambique, Namibia, Nigeria, Senegal, Swaziland, Togo, Uganda, Zambia, Zimbabwe
Chemicals		Equatorial Guinea, Ghana, South Africa, Mozambique, Niger, Rwanda	Benin, Botswana, Burkina Faso, Burundi, Cameroon, Congo Dem. Rep., Congo Rep., Cote d'Ivoire, Djibouti, Ethiopia, Gabon, Kenya, Madagascar, Malawi, Mali, Mauritius, Namibia, Nigeria, Senegal, Somalia, Swaziland, Sudan, Togo, Uganda, Zambia, Zimbabwe
Leather products		Rwanda, Senegal, Cabo Verde, Kenya, Mauritius, South Africa	Burundi, Cote d'Ivoire, Ethiopia, Lesotho, Namibia, Nigeria, Madagascar, Mali, Ghana, Somalia, Sudan, Uganda, Zambia, Zimbabwe
Basic manufactures	South Africa	Congo Dem. Rep., Congo Rep., Ethiopia, Gabon, Burkina Faso, Madagascar, Mozambique, Namibia, Zambia	Benin, Botswana, Burundi, Cameroon, Cote d'Ivoire, Ghana, Kenya, Liberia, Malawi, Mauritius, Nigeria, Rwanda, Senegal, Swaziland, Sudan, Togo, Uganda, Zimbabwe
Non-electronic machinery		Benin, Burkina Faso, Cameroon, South Africa, Swaziland, Rwanda, Togo	Botswana, Congo Dem. Rep., Congo Rep., Cote d'Ivoire, Equatorial Guinea, Gabon, Ghana, Guinea, Kenya, Liberia, Madagascar, Malawi, Mali, Mauritius, Mozambique, Namibia, Niger, Nigeria, Senegal, Sierra Leone, Sudan, Uganda, Zambia, Zimbabwe
IT and Consumer electronics		Botswana, Cote d'Ivoire, Kenya, Mali, Madagascar, Mauritius, Rwanda, South Africa	Congo Dem. Rep., Gabon, Ghana, Lesotho, Namibia, Niger, Malawi, Senegal, Uganda, Zambia, Zimbabwe
Electronic components		Botswana, Gabon, Liberia, Madagascar, South Africa, Swaziland	Cameroon, Congo Rep., Cote d'Ivoire, Ghana, Kenya, Lesotho, Malawi, Mali, Mauritius, Namibia, Niger, Senegal, Uganda, Zambia, Zimbabwe
Transport equipment		Benin, Burkina Faso, Mauritius, South Africa, Senegal, Uganda	Botswana, Cameroon, Congo Dem. Rep., Congo Rep., Cote d'Ivoire, Equatorial Guinea, Gabon, Ghana, Guinea, Kenya, Liberia, Madagascar, Malawi, Mali, Mozambique, Namibia, Niger, Rwanda, Seychelles, Sierra Leone, Swaziland, Sudan, Togo, Zambia, Zimbabwe
Clothing	Mauritius	Cote d'Ivoire, Ethiopia, Ghana, Lesotho, Madagascar, Senegal, South Africa, Swaziland, Uganda, Zimbabwe	Botswana, Cabo Verde, Eritrea, Kenya, Malawi, Namibia
Miscellaneous manufacturing		Benin, Botswana, Burkina Faso, Congo Dem. Rep., Ethiopia, Ghana, Liberia, Mauritius, Senegal, Seychelles, South Africa, Togo	Congo Rep., Cote d'Ivoire, Gabon, Guinea, Kenya, Lesotho, Madagascar, Malawi, Mali, Mozambique, Namibia, Niger, Nigeria, Rwanda, Sierra Leone, Swaziland, Sudan, Uganda, Cameroon, Zambia, Zimbabwe
Minerals	Botswana, Equatorial Guinea, Eritrea, South Africa, Rwanda,	Angola, Burkina Faso, Chad, Congo Dem. Rep., Congo Rep., Gabon, Ghana, Guinea, Kenya, Lesotho, Liberia, Madagascar, Mauritania, Mauritius, Mozambique, Namibia, Senegal, Nigeria, Swaziland, Sudan	Benin, Burundi, Cameroon, Cote d'Ivoire, Ethiopia, Malawi, Mali, Seychelles, Togo Uganda, Zambia, Zimbabwe

	Niger		

Table 12: Distribution of number of Sub-Saharan African Countries by Sector and Cluster (Second Set of Criteria)

Sector	Country clusters				Countries in total	$I_{sec.}$	Sectoral clusters
	Cluster 1	Cluster 2	Cluster 3	Cluster 4			
Fresh food		4	26	15	45	7.9	3
Processed food		1	13	24	38	5.3	3
Wood products		1	14	14	29	4.5	4
Textiles			4	19	23	2.7	4
Chemicals			6	26	32	3.8	4
Leather products			6	14	20	2.6	4
Basic manufactures		1	9	18	28	3.9	4
Non-electronic machinery			7	24	31	3.8	4
IT and Consumer electronics			8	11	19	2.7	4
Electronic components			6	15	21	2.7	4
Transport equipment			6	25	31	3.7	4
Clothing		1	10	6	17	2.9	4
Miscellaneous manufacturing			12	21	33	4.5	4
Minerals		6	20	12	38	7	3

Table 13: Superior and Inferior Positions of Sub-Saharan African Countries in Export Sectors of Economy (Second Set of Criteria)

Sector	Superior positions	Inferior positions
Fresh food	Cote d'Ivoire (11), South Africa (11), Kenya (10), Uganda (10), Burkina Faso (9), Ethiopia (9), Madagascar (9), Seychelles (9), Sierra Leone (9)	Niger (2), Swaziland (2), Equatorial Guinea (1), Gabon (0)
Processed food	South Africa (11), Seychelles (7), Swaziland (7), Rwanda (7), Malawi (6),	Zambia (3), Cote d'Ivoire (3), Namibia (3), Ghana (3), Madagascar (3), Benin (2), Burkina Faso (2), Djibouti (2), Cameroon (2), Mozambique (2), Nigeria (2), Zimbabwe (2), Congo Dem. Rep. (1), Congo Rep., (1), Lesotho (1), Mali (1), Sudan (1), Botswana (1),
Wood products	South Africa (10), Equatorial Guinea (9), Congo Rep. (9), Cameroon (8)	Botswana (3), Ethiopia (3), Madagascar (3), Nigeria (3), Zimbabwe (3), Namibia (2), Lesotho (2), Malawi (1), Benin (1),
Textiles	South Africa (8), Madagascar (5), Mauritius (5), Niger (5), Nigeria (4), Cameroon (4)	Ethiopia (3), Ghana (3), Mozambique (3), Namibia (3), Senegal (3), Swaziland (3), Zimbabwe (3) Burkina Faso (2), Zambia (2), Benin (1), Botswana (1), Cote d'Ivoire (1), Gambia (1), Kenya (1), Lesotho (1), Togo (1)
Chemicals	South Africa (9), Mozambique (6), Rwanda (5), Niger (5), Ghana (5), Equatorial Guinea (5)	Benin (3), Burundi (3), Kenya (3), Gabon (3), Zimbabwe (3), Cameroon (2), Congo Dem. Rep. (2), Congo Rep. (2), Cote d'Ivoire (2), Ethiopia (2), Malawi (2), Zambia (2), Senegal (2), Mali (1), Namibia (1), Nigeria (1), Swaziland (1), Sudan (1), Togo (1), Somalia (0)
Leather products	South Africa (8), Mauritius (6), Senegal (5), Rwanda (5), Cabo Verde (5)	Burundi (3), Sudan (3), Uganda (3), Zimbabwe (3), Cote d'Ivoire (3), Nigeria (3), Lesotho (2), Madagascar (2), Mali (2), Namibia (2), Somalia (1)
Basic manufactures	South Africa (10), Zambia (8), Madagascar (8), Congo Rep. (7), Burkina Faso (7)	Botswana (3), Burundi (3), Liberia (3), Rwanda (3), Togo (3), Uganda (3), Zimbabwe (3), Swaziland (3), Cote d'Ivoire (2), Senegal (2), Malawi (2), Mauritius (2), Sudan (1),

Non-electronic machinery	South Africa (8), Rwanda (7), Swaziland (6), Benin (6)	Congo Rep. (2), Cote d'Ivoire (2), Sierra Leone (2), Equatorial Guinea (2), Ghana (2), Mauritius (2), Guinea (2), Kenya (2), Niger (2), Malawi (1), Mali (1), Mozambique (1), Sudan (1), Nigeria (0)
IT and Consumer electronics	South Africa (7), Mali (6), Madagascar (6), Botswana (5), Cote d'Ivoire (5), Kenya (5), Mauritius (5), Rwanda (5)	Zambia (4), Zimbabwe (4), Namibia (4), Senegal (4), Congo Dem. Rep. (3), Lesotho (3), Malawi (3), Niger (2), Uganda (2), Gabon (2), Ghana (2)
Electronic components	South Africa (8), Swaziland (7), Madagascar (7), Liberia (7), Botswana (7)	Mauritius (2), Niger (2), Senegal (2), Zambia (2), Zimbabwe (2) Congo Rep.(1), Cote d'Ivoire (1), Kenya (1), Malawi (1), Ghana (0)
Transport equipment	Benin (7), Uganda (7), South Africa (6), Burkina Faso (5), Mauritius (5), Senegal (5)	Togo (2), Zambia (2), Rwanda (2), Cameroon (1), Malawi (1), Seychelles (1), Sudan (1), Zimbabwe (1), Congo Dem. Rep. (0), Sierra Leone (0)
Clothing	Mauritius (10), Swaziland (8), Uganda (8), Lesotho (8), Madagascar (8), South Africa (7)	Kenya (3), Namibia (3), Botswana (2), Malawi (2), Eritrea (1)
Miscellaneous manufacturing	South Africa (8), Ghana (7), Togo (6), Senegal (6), Ethiopia (6), Burkina Faso (6), Benin (6)	Congo Rep. (2), Cote d'Ivoire (2), Namibia (2), Niger (2), Nigeria (2), Sierra Leone (2), Zambia (2), Zimbabwe (2), Guinea (2), Madagascar (2), Malawi (1), Uganda (1), Gabon (1), Sudan (0)
Minerals	Botswana (12), Niger (12), South Africa (12), Eritrea (11), Rwanda (11), Equatorial Guinea (10),	Benin (3), Burundi (3), Ethiopia (3), Seychelles (3), Zimbabwe (3), Cameroon (2), Mali (1)

Since in this case the criteria are twice as stringent, some countries, when comparing to the information in Tables 7 and 9, move to less competitive country clusters (Tables 12, 13), and a number of sectoral clusters move to worse positions (Table 12). South Africa in all sectors was among the countries in the superior positions (Table 13), while Malawi, as in the previous case (Table 10), had 9 worst positions out of 14.

The procedure for automatic calculation of threshold criteria needs to be further developed. It can include using the K-Means clustering algorithm with dividing the set of indicator values into two clusters, when the new values of one of the clusters are assigned the zero value and those of the other one are assigned the one value.

It should be noted that clustering binary matrices of the matrix type shown in Table 4 can also be carried out in terms expressed in [1-8], that is, by sorting out dense submatrices consisting of ones in them.

III. CONCLUSION

Thus, in this paper a matrix clustering methodology has been proposed, which involves constructing the initial state matrix of objects, multicriteria threshold binarization of this matrix, and clustering the obtained binary matrix into submatrices with different densities of zero or one elements. Using hand computation, this methodology has been tested on the indicators of export competitiveness of all Sub-Saharan African countries for Fresch Food sector from the Trade Competitiveness Map data. A standard R program has been developed for multicriteria threshold binarization and clustering arbitrary state matrices, and calculations have been made for all 14 export sectors of Sub-Saharan African economies, using the data from the Trade Competitiveness Map for two sets of criteria. The program has been fine-tuned on the example of hand computation for the Fresh Food sector. The procedure for selecting threshold criteria values is proposed to be automated by using the K-Means clustering algorithm for two clusters consisting of zeros and ones.

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