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MODIFIED SCHEME OF CRYSTALLINE UNDULATOR

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The new scheme of crystalline undulator based on coherent bremsstrahlung from above barrier relativistic electrons interacting with a system of atomic strings periodically deformed by acoustic wave is proposed and studied in the work. The possibility to generate X-rays in the range of about 10 keV and more by electrons with energies of about 100 MeV is shown. Spectral-angular density of the emission being considered may be six times greater than ordinary CB.

Keywords: Relativistic electron; coherent bremsstrahlung; periodically deformed atomic strings; emission spectral-angular distribution.

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1. Introduction

Coherent emission from relativistic electrons moving through an aligned crystal is of interest from the viewpoint of quasi-monochromatic X and gamma quanta producing on the base of such emission mechanisms as coherent bremsstrahlung (CB),[1] channeling radiation,[2,3] parametric X radiation.[4] Currently, much attention is given to the problem of gamma source based on the crystalline undulator (see Ref. 5 and references therein). The traditional approach to the realization of the source under discussion suggests an emission from relativistic positrons channeling in a system of periodically deformed atomic planes of the crystal. The main disadvantage of this scheme consists of a small achievable length of such source caused by dechanneling processes. Another way to realize gamma-ray source based on the crystalline undulator consists of the use of the above barrier particle emission in the potential of periodically deformed atomic planes in the crystal.[6] Such a scheme has no need of an emitting particle channeling and hence one can increase the emission yield substantially using electron beam instead of low-power positron beam. It should be noted in this connection that CB from relativistic electrons moving in a crystal with periodically deformed crystalline lattice was studied before within the frame of a more general problem of controlling the characteristics of electromagnetic processes in crystals by an influence of external fields, such as acoustic waves, temperature gradient, etc.[7–11] Performed studies have shown that the interaction of fast electrons with deformed atomic strings allows one to realize more intensive emission than that with deformed atomic planes[11] and so CB from relativistic electrons on a system of periodically deformed atomic strings is the subject of our study. The aim of the paper is to show a possibility to realize the new scheme of crystalline undulator, permitting one to produce intensive quasi-monochromatic X-rays in the range of about 10 keV and more by electron beams with energies of the order of 100 MeV.

2. The Emission Spectral-Angular Distribution

Let us consider bremsstrahlung from relativistic electrons moving in a crystalline target nearly parallel to one of the main crystallographic axes. Since the emission of photons with energies ω much less than the energy ϵ_0 of an emitting electron is of interest in the process being studied classical electrodynamics[12] may be used for the determination of the emission amplitude $\mathbf{A}$

$$\mathbf{A} = \frac{i\omega e}{2\pi} \int dt \left(\mathbf{V}(t) - \frac{\mathbf{n}}{\sqrt{\epsilon}} \right) \exp(i\omega(t - \sqrt{\epsilon}\mathbf{n} \cdot \mathbf{r}(t))) \,, \tag{1}$$

where $\mathbf{V} = d\mathbf{r}/dt$, $\mathbf{r}(t)$ is the trajectory of an emitting electron, ϵ is the target dielectric permeability, and $\mathbf{n}$ is the unit vector to the direction of an emitted photon propagation. Supposing the characteristic angle of electron scattering by atomic string to be less than $\gamma^{-1} = m/\epsilon_0$, we will use the approximation of rectilinear trajectory $\mathbf{r}(t) = \mathbf{V}t + \mathbf{r}_0$ to calculate the integral (1). The result of calculations can be expressed in terms of the potential of target atoms

$$\begin{aligned} \mathbf{A} = {} & \frac{ie^3}{2\pi^2\epsilon_0} \frac{1}{1 - \sqrt{\epsilon_0}\mathbf{n} \cdot \mathbf{V}} \int \frac{d^3k}{k^2} (Z - F(k)) \\ & \times \sum_n \exp(i\mathbf{k} \cdot (\mathbf{r}_n - \mathbf{r}_0)) \mathbf{T}(\mathbf{k}) \delta(\omega(1 - \sqrt{\epsilon}\mathbf{n} \cdot \mathbf{V}) - \mathbf{k} \cdot \mathbf{V}) \end{aligned} \tag{2}$$

$$\mathbf{T}(\mathbf{k}) = \mathbf{k} - \mathbf{V}(\mathbf{k} \cdot \mathbf{V}) - \frac{\mathbf{n} - \sqrt{\epsilon}\mathbf{V}}{1 - \sqrt{\epsilon}\mathbf{n} \cdot \mathbf{V}} (\mathbf{n} \cdot \mathbf{k} - (\mathbf{n} \cdot \mathbf{V})(\mathbf{k} \cdot \mathbf{V})) \,.$$

Here Z is the atomic number, $F(k)$ is the form factor of an atom, $\mathbf{r}_n$ is the coordinate of nth atom, and the summation is over all atoms in the target.

Spectral-angular distribution of the number of emitted photons is determined be the general formula[12]

$$\omega \frac{dN}{d\omega d\Omega} = \langle |A|^2 \rangle \,, \tag{3}$$

where the brackets $\langle\,\rangle$ mean the averaging over both atomic coordinates $\mathbf{r}_n$ in the target and $\mathbf{r}_0$ in the limits of an elementary cell.

Performing the averaging over $\mathbf{r}_n$, one should take into account the periodical displacement of atoms in a crystalline lattice (for simplicity sake, assume that only one atom is placed in an elementary cell) and the perturbation of the crystalline lattice by acoustic wave, so that the coordinate $\mathbf{r}_n$ may be presented in the form

$$\mathbf{r}_n = \mathbf{r}_n + \mathbf{u}_n + \mathbf{a}\sin(\boldsymbol{\xi}\cdot\mathbf{r}_n), \tag{4}$$

where $\mathbf{r}_n$ is the equilibrium position of nth atom in the lattice, $\mathbf{u}_n$ is its thermal displacement, $\mathbf{a}$ and $\boldsymbol{\xi}$ are the amplitude and wave vector of the acoustic wave respectively. With account of (4), one can obtain from (3) the following expression for the emission density:

$$\begin{aligned}\langle|A|^2\rangle &= \frac{e^6}{4\pi^4\epsilon_0^2}\frac{1}{(1-\sqrt{\epsilon}\mathbf{n}\cdot\mathbf{V})^2}\int\frac{d^3k_1}{k_1^2}\frac{d^3k_2}{k_2^2}(Z-F(k_1))(Z-F(k_2))\mathbf{T}(\mathbf{k}_1)\mathbf{T}(\mathbf{k}_2)\\ &\times\left[(2\pi)^3 n_0\left(\exp\left(-\frac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)^2u_T^2\right)-\exp\left(-\frac{1}{2}(k_1^2+k_2^2)^2u_T^2\right)\right)\right.\\ &\times\sum_g\sum_l J_l((\mathbf{k}_1-\mathbf{k}_2)\mathbf{a})\delta(\mathbf{k}_1-\mathbf{k}_2+l\boldsymbol{\xi}+\mathbf{g})+(2\pi)^6n_0^2\\ &\times\exp\left(-\frac{1}{2}(k_1^2+k_2^2)^2u_T^2\right)\sum_{g_1}\sum_{g_2}\sum_l\sum_p J_l(\mathbf{k}_1\cdot\mathbf{a})J_p(\mathbf{k}_2\cdot\mathbf{a})\\ &\times\left.\delta(\mathbf{k}_1+l\boldsymbol{\xi}-\mathbf{g}_1)\delta(\mathbf{k}_2+p\boldsymbol{\xi}-\mathbf{g}_2)\right]\delta(\omega(1-\sqrt{\epsilon}\mathbf{n}\cdot\mathbf{V})-\mathbf{k}_1\cdot\mathbf{V})\\ &\times\delta(\omega(1-\sqrt{\epsilon}\mathbf{n}\cdot\mathbf{V})-\mathbf{k}_2\cdot\mathbf{V})\langle e^{(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{r}_0}\rangle\,,\end{aligned}\tag{5}$$

where u_T is the mean square amplitude of thermal vibrations of an atom, $\mathbf{g}_{1,2}$ are reciprocal lattice vectors, and n_0 is the density of atoms.

The first item in square brackets (5) corresponds to incoherent contribution of target atoms to the formation of bremsstrahlung yield. The second one describes the coherent contribution of interest to us. The result of averaging depends strongly on the coefficient $\langle\exp((\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{r}_0)\rangle$. Since the inequality

$$l_{\text{eff}}\xi a \ll 1\,, \tag{6}$$

is valid for all practical purposes in the case of an acoustic wave propagating in a crystal, one can obtain the following estimation:

$$\langle\exp((\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{r}_0)\rangle = \frac{1}{\Omega_0}\int_{\Omega_0}d^3r_0\exp(i(\mathbf{g}-l\boldsymbol{\xi})\cdot\mathbf{r}_0) = \begin{cases}1 & \text{if } g=0\\ (l\xi/g)^3\ll 1 & \text{if } g\neq 0\end{cases}, \tag{7}$$

where Ω_0 is the volume of the crystal elementary cell. Based on the formula (7), one can substantially reduce the result (5) and obtain the simple formula for the spectral-angular distribution of the intensity of coherent part of bremsstrahlung

$$\omega\frac{dN^{coh}}{dtd\omega d\Omega}=\frac{8\pi Z^2e^6n_0^2}{\epsilon_0^2}\frac{1}{(1-\sqrt{\epsilon}\mathbf{n}\cdot\mathbf{V})^2}\sum_{\mathbf{g}}\sum_{l}\frac{\exp(-g_l^2u_T^2)}{(g_l^2+R^{-2})^2}J_l^2(\mathbf{g}_l\cdot\mathbf{a})T^2(\mathbf{g}_l)$$
$$\times\,\delta(\omega(1-\sqrt{\epsilon}\mathbf{n}\cdot\mathbf{V})-\mathbf{g}_l\cdot\mathbf{V})\,,\tag{8}$$

where $\mathbf{g}_l=\mathbf{g}-l\boldsymbol{\xi}$, the simplest statistical model of an atom with exponential screening is used in Eq. (8), where R is the screening radius in the Fermi–Thomas model.

The result (8) is the basic one for further analysis. Obviously, in the limiting case $\mathbf{a}\to 0$, only item with index $l=0$ takes the contribution to the sum (8). As this takes place, the result (8) is reduced to well known formula for traditional coherent bremsstrahlung from relativistic electrons moving in a crystal with rectilinear atomic strings.

3. Mechanism of Coherent Bremsstrahlung Enhancement by Acoustic Wave in Conditions of Incident Electron Interaction with a Gas of Deformed Atomic String

Let us use the result (8) for studies of an influence of acoustic wave on CB characteristics. It is convenient for further analysis to simplify Eq. (8) with account of the property (6), allowing one to neglect the difference between $\mathbf{g}$ and $\mathbf{g}_1$ in the argument of Bessel function in Eq. (8). Wavelength of acoustic wave exceeds essentially the distance between atoms in the target and hence the above difference is not substantial throughout this formula except the argument of δ-function.

Since the coherent contribution of string atoms to the formation of bremsstrahlung yield provides the basis for the emission mechanism in question, one should reserve in the sum $\sum_{\mathbf{n}}$ in Eq. (8) summation over transversal relative to string axis $\mathbf{e}_x$ components of reciprocal lattice vectors $\mathbf{g}_\perp$ only. Introducing the angular variables $\boldsymbol{\theta}$ and $\boldsymbol{\psi}$ by the expressions

$$\mathbf{V}=\mathbf{e}\left(1-\frac{1}{2}\gamma^{-2}\right),\quad \mathbf{n}=\mathbf{e}\left(1-\frac{1}{2}\theta^2\right)+\boldsymbol{\theta},\quad \mathbf{e}=\mathbf{e}_x\left(1-\frac{1}{2}\psi^2\right)+\boldsymbol{\psi},\quad \mathbf{e}_x\cdot\boldsymbol{\psi}=0\,.\tag{9}$$

One can bring Eq. (8) into the form

$$\omega\frac{dN^{coh}}{dtd\omega d^2\theta}=\frac{32\pi Z^2e^6n_0^2}{m^2\gamma^2(\gamma_*^{-2}+\theta^2)^2}\sum_{g_\perp,l}\frac{\exp(-g_\perp^2u_T^2)}{(g_\perp^2+R^{-2})^2}J_l^2(\mathbf{g}_\perp\cdot\mathbf{a}_\perp)$$
$$\times\left(g_\perp^2-\frac{4\gamma_*^{-2}(\boldsymbol{\theta}\cdot\mathbf{g}_\perp)^2}{(\gamma_*^{-2}+\theta^2)^2}\right)\delta\left(\frac{\omega}{2}(\gamma_*^{-2}+\theta^2)+l\xi_x-\mathbf{g}_\perp\cdot\boldsymbol{\psi}\right),\tag{10}$$

where $\gamma_*^{-2}=\gamma^{-2}+\omega_0^2/\omega^2$, γ is the Lorentz factor, ω_0 is the plasma frequency of the target, the ordinary approximation for dielectric permeability is used $\epsilon=1-\omega_0^2/\omega^2$.

Obviously, the emission intensity (10) is increased with decreasing of the orientation angle ψ between string axis and the velocity direction of an emitting electron. On the other hand, there is a strong azimuth scattering of a projectile on a string potential in the range of small orientation angles $\psi \approx \psi_{ch}$ (ψ_{ch} is the channeling angle), which is why the correlations between consecutive collisions of a fast electron with atomic strings are destroyed and hence such collisions will become accidental. Summation over $\mathbf{g}_\perp$ in Eq. (10) can be replaced by integration in conditions under consideration $\sum_{g_\perp} \to (d_\perp/2\pi)^2 \int d^2 g_\perp$, where $d_\perp$ is the average distance between atomic strings. The result of integration has the simple form

$$\omega \frac{dN^{coh}}{dtd\omega d^2\theta} = \frac{8\pi Z^2 e^6 n_0^2}{m^2\gamma^2} \frac{R}{d_x \psi} \frac{\gamma_*^{-4} + \theta^4}{(\gamma_*^{-2} + \theta^2)^4} \sum_l J_l^2(\eta_l) \frac{1 + 2\eta_l^2 (R/a_\perp)^2}{(1 + \eta_l^2 (R/a_\perp)^2)^{3/2}},$$
$$\eta_l = \frac{\omega a_\perp}{2\psi}(\gamma_*^{-2} + \theta^2) + l\frac{\xi_x a_\perp}{\psi}. \tag{11}$$

Thermal vibrations of crystal atoms are neglected in the formula (11) describing CB from relativistic electrons moving in a gas of deformed atomic strings (the characteristic coefficient $R/d_x\psi$ shows the number of string atoms making coherent contribution to the formation of CB yield on a single string, d_x is the distance between neighboring atoms in the string). Formula (11) allows one to describe an influence of acoustic wave on CB spectral-angular properties. Let us consider the orientation-dependence of strongly collimated CB. In conditions $\theta^2 \ll \gamma_*^{-2}$ under consideration, the formula (11) can be presented in the form

$$\omega \frac{dN^{coh}}{dtd\omega d^2\theta} = \frac{8\pi Z^2 e^6 n_0^2}{m^2}\gamma^2 \frac{R}{d_x \psi_*} F\left(x, \beta, \tau, l\frac{\psi_*}{\psi}\right),$$

$$F = \left(\frac{x}{x + x^{-1}}\right)^2 \frac{\psi_*}{\psi} \sum_l J_l^2\left(\beta(x + x^{-1}) + l\frac{\psi_*}{\psi}\right) \frac{1 + 2\tau^2\left(\beta(x + x^{-1}) + l\frac{\psi_*}{\psi}\right)^2}{\left(1 + \tau^2\left(\beta(x + x^{-1}) + l\frac{\psi_*}{\psi}\right)^2\right)^{3/2}}, \tag{12}$$

much convenient for numerical calculations, where $x = \omega/(\gamma\omega_0)$, $\beta = \omega_0 a_\perp/(2\gamma\psi)$, $\psi_* = \xi_x a_\perp$, $\tau = R/a_\perp$. The universal function $F(x)$ describing the form of CB spectrum has been calculated for fixed values of the parameters β and τ and different values of the parameter $\psi_*/\psi < 1$. Results of calculations are demonstrated in Fig. 1.

Curve 1 corresponds to an ordinary CB in the potential of rectilinear strings. Curves 2 and 3 have been calculated for growing values of the ratio ψ_*/ψ (amplitude of acoustic wave $a_\perp$ was chosen to be of the order of the distance between atoms, and the orientation angle ψ used in calculations was about γ^{-1} and $\gamma \sim 100$). Presented curves 2 and 3 demonstrate the growth of collimated CB yield for $\psi \to \psi_*$. To elucidate this phenomenon let us consider the same orientation dependence but in the range $\psi < \psi_*$. Corresponding curves are presented in Fig. 2.

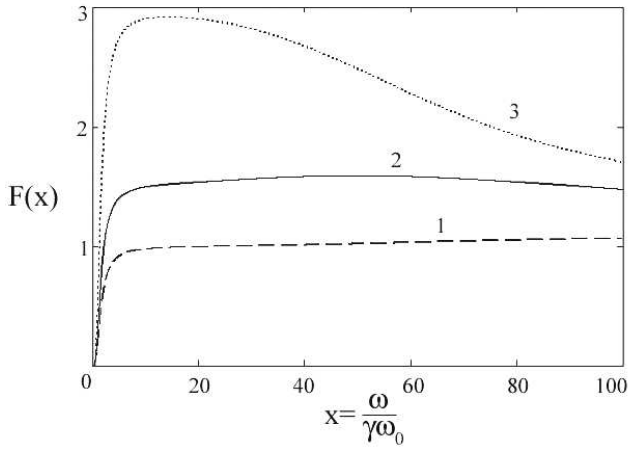

Fig. 1. Curves 2 and 3 have been calculated for fixed values of the parameters $\beta = 0.05, \tau = 0.1$ and different values of the parameter $\psi_*/\psi = 0.75$ (curve 2) and $\psi_*/\psi = 0.95$ (curve 3).

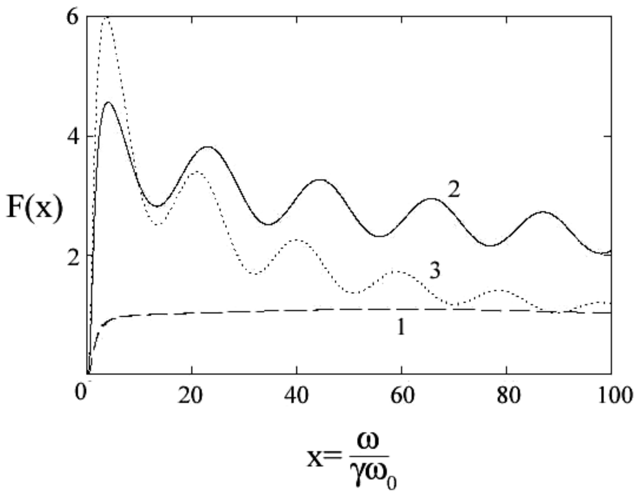

Fig. 2. The same but for $\psi < \psi_*$, fixed parameters $\beta = 0.24, \tau = 0.05$ and different values of the parameter $\psi_*/\psi = 4$ (curve 2) and $\psi_*/\psi = 1.1$ (curve 3).

According to above figure, the yield peaks in the vicinity of $\psi = \psi_*$ as well, but achieved enhancement coefficient is more than that in conditions $\psi > \psi_*$ by a factor of 2. In addition to this, the strong oscillations appear in CB spectrum in conditions $\psi < \psi_*$ being considered.

The indicated properties are caused by the mechanism of emission enhancement. The greatest contribution to electron CB yield is formed at the parts of its trajectory, parallel to the local direction of bended string axis. Such interaction is not possible under conditions of CB in the potential of rectilinear strings (CB cross-section increases without limits when $\psi \to 0$) and hence it is this case of periodically deformed strings that offers the realization of necessary conditions. It is clear, that the length of above parts of an emitting electron trajectory peaks in the vicinity of $\psi = \psi_*$. (Obviously, the angle ψ_* is the maximum angle of a string axis bending.) That is the reason that the maximum of CB yield is achieved when $\psi = \psi_*$. One can readily see that an emitting electron has only one intersection with a single string in the range $\psi > \psi_*$, which is why the corresponding emission spectrum has only one maximum. (Maximum is formed due to the influence of Ter-Mikaelian effect of dielectric suppression.) On the other hand, there are several above-intersections in the range $\psi < \psi_*$. As a consequence, an interference between elementary waves emitted from different parts of electron trajectory corresponding to different intersections is responsible for oscillations in the emission spectrum.

4. Conclusion

The new scheme of crystalline undulator has been proposed and studied in the present work. Within the frame of the scheme being proposed, relativistic electron beam is directed at a small angle to the axis of a system of atomic strings, periodically deformed by an acoustic wave. Orientation angle between an emitting electron velocity and string axis is chosen to be small enough so that the electron CB on a single string reaches its maximum, but collisions of incident electron with different strings become accidental. In conditions under consideration, the main contribution to the formation of CB yield is made by the parts of fast electron trajectory parallel to the local direction of bended string axis. The length of such parts achieves its maximum when the orientation angle ψ between electron velocity and string axis is equal to the maximum angle of a string axis bending ψ_*.That is the reason that the maximum of CB yield is achieved when $\psi = \psi_*$.

The emitting electron has only one intersection with a single string in the range $\psi > \psi_*$, which is why the corresponding emission spectrum has only one maximum formed due to the influence of Ter-Mikaelian effect of dielectric suppression. On the other hand, there are several above-intersections in the range $\psi < \psi_*$ and hence strong oscillations appear in the CB spectrum due to an interference between different parts of electron trajectory corresponding to different intersections.

Performed calculations have shown that the proposed scheme of crystalline undulator allows one to realize CB spectral-angular density which is six times greater than that in conditions of ordinary CB on rectilinear strings.

The emission mechanism being considered can be realized for electron energies of the order of 100 MeV. As this takes place, intense X-rays can be generated in the range of about 10 keV and more.

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