Effect of Internal Heat Evolution on the Motion of a Solid Particle in a Viscous Fluid

N. V. Malay*, A. A. Pleskanev*, and E. R. Shchukin**

* *Belgorod State University, Belgorod, 308015 Russia*
e-mail: malay@bsu.edu.ru
** *Institute of High Temperatures Scientific Association (IVTAN), Russian Academy of Sciences, Izhorskaya ul. 13/19, Moscow, 127412 Russia*

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Abstract—The problem of the effect of internal heat evolution on the motion of a heated solid spherical particle in a viscous fluid is analytically solved in the Stokes approximation at small Reynolds and Peclet numbers. The temperature drop between the surface of the particle and the area away from it is assumed to be arbitrary. In solving hydrodynamic equations, the thermal conductivity of the particle is set to be a power function of temperature and the viscosity of the fluid, an exponential–power function of temperature. The observability of this effect is discussed.

INTRODUCTION

We consider the steady motion of a solid nonuniformly heated hydrosol particle of radius R in a viscous incompressible fluid occupying the whole space. The particle is heated by internal heat sources nonuniformly distributed over its volume with density $q_i(r, \theta)$, where r and θ are spherical coordinates ($0 \le \theta \le \pi$). Heating of a hydrosol particle may be associated with a chemical reaction proceeding in its volume, its radioactive decay, electromagnetic radiation absorption, etc. As a result, the surface temperature of the particle may far exceed the environmental temperature. This, in turn, may change the electrophysical properties of the fluid and, thereby, substantially affect the velocity and pressure field distributions in the neighborhood of the particle. Of all fluid transport coefficients, the dynamic viscosity is the most temperature-dependent [1]. This dependence is given by the formula

$$\mu_e = \mu_\infty\left[1 + \sum_{n=1}^{\infty} F_n\left(\frac{T_e}{T_\infty} - 1\right)^n\right]\exp\left\{-A\left(\frac{T_e}{T_\infty} - 1\right)\right\}, \quad (1)$$

which describes the variation of the viscosity in a wide temperature range (at $F_n = 0$, this formula can be reduced to the well-known Reynolds formula [1]). Here, A and F_n are constants, $\mu_\infty = \mu_e(T_\infty)$, and T_∞ is the fluid temperature away from the particle. For water, we have $A = 5.779$, $F_1 = -2.318$, $F_2 = 9.118$, and $T_\infty = 273$ K; for glycerol, $A = 17.29$, $F_1 = -1.228$, $F_2 = 7.022$, and $T_\infty = 303$ K. The relative error of formula (1) is no higher than 3% in both cases. Factors F_n were calculated using the Maple VIII software package. Hereafter, indices "e," "i," and "∞" refer, respectively, to external parameters (those of the viscous fluid), internal parameters (those of the particle), and parameters at infinity (away from the particle).

Interacting with the nonuniformly heated surface, the fluid starts moving over the surface toward higher temperatures. This effect, called the thermal slip, causes additional force $\mathbf{F}_q$ to arise. Once this force becomes equal to the viscous force of the environment, the particle starts moving uniformly. Thus, if the temperature drop near the particle is significant (i.e., $(T_{\text{is}} - T_\infty)/T_\infty \sim O(1)$, where T_{is} is the mean temperature of the particle surface in the viscous fluid), the solid particle moves steadily.

When the particle is heated through electromagnetic radiation absorption, its steady motion in a fluid is termed photophoresis [2, 3]. In this case, the tangential component of the velocity on the surface of a solid aerosol particle ($r = R$) satisfies the slip condition [4],

$$U_\theta = K_{\text{ts}}\frac{\nu_e}{RT_e}\frac{\partial T_e}{\partial \theta}. \quad (2)$$

Here, U_θ is the shear component of mass velocity $\mathbf{U}$ in the spherical coordinate system; ν_e is the dynamic viscosity of the fluid; T_e is the fluid temperature; and K_{ts} is the thermal slip coefficient, which is derived from the kinetic theory of gases. If the accommodation coefficients for the tangential momentum, α_τ, and energy, α_E, equal unity, gaskinetic coefficient $K_{\text{ts}} = 1.152$ [4].

The problem of fluid slip over the surface of a solid hydrosol particle was first solved by Basset [5]. He supposed that the tangential velocity of a fluid relative to the surface of a solid is proportional to shear stresses and named the corresponding proportionality coefficient (K_{ts}) the slip coefficient. It is assumed that this coefficient depends on only the nature of the fluid and solid surface (if it is other than zero). If the spherical particle is at rest and the fluid flows about it, this hypothesis for the axisymmetric flow (the Basset hypothesis) takes the form

$$U_e = K_{ts}\mu_e\left(\frac{\partial U_\theta}{\partial r} + \frac{1}{r}\frac{\partial U_r}{\partial \theta} - \frac{U_\theta}{r}\right).$$

Elaboration of a theory accounting for the effect of internal heat evolution on the motion of a solid spherical particle in a viscous nonisothermal fluid is a challenging task. First, the motion of a particle in a fluid is governed by both surface and volume effects arising from nonuniform distributions of hydrodynamic and temperature fields. Second, this motion is a result of slip of the fluid over the solid surface. Thermal slip of a fluid over a solid surface is not yet fully understood. The reason is that a rigorous mathematical theory of inhomogeneous fluids is still lacking. Third, a mechanism of thermal energy distribution inside the solid that causes its nonuniform heating is unknown. However, impressive theoretical and experimental advances in this respect have recently been made [6, 7].

1. PROBLEM DEFINITION

We consider the steady motion of a solid nonuniformly heated spherical hydrosol particle of radius R in a viscous incompressible fluid occupying the whole space. The particle is heated by heat sources nonuniformly distributed over its volume with density $q_i(r, \theta)$. It is assumed that any phase transition is absent, the thermal conductivity of the particle far exceeds the thermal conductivity of the fluid, and the thermal conductivity of the particle is a power function of temperature ($\lambda_i = \lambda_{i\infty} t_i^{\omega}$). Here, $\lambda_{i\infty} = \lambda_i(T_\infty)$ and $t_i = T_i/T_\infty$. The particle moves in the spherical coordinate system (r, θ, φ) with the origin at its center.

In view of the above assumptions, the equations and boundary conditions for velocity $\mathbf{U}_e$, pressure P_e, and temperatures T_e and T_i in the spherical coordinate system have the form [8, 9],

$$\nabla P_e = \mu_e \Delta \mathbf{U}_e + 2(\nabla\mu_e \cdot \nabla)\mathbf{U}_e + [\nabla\mu_e \times \operatorname{curl}\mathbf{U}_e]; \quad \operatorname{div}\mathbf{U}_e = 0; \tag{1.1}$$

$$\Delta T_e = 0; \tag{1.2}$$

$$\operatorname{div}(\lambda_i \nabla T_i) = -q_i; \tag{1.3}$$

$$r = R\colon U_r = 0, \quad U_\theta = K_{ts}\frac{\nu_e}{RT_e}\frac{\partial T_e}{\partial \theta}, \quad T_e = T_i,$$
$$\lambda_e \frac{\partial T_e}{\partial r} = \lambda_i \frac{\partial T_i}{\partial r} + \sigma_0\sigma_1(T_i^4 - T_\infty^4),$$
$$r \longrightarrow \infty\colon \mathbf{U}_e \longrightarrow U_\infty \cos\theta \mathbf{e}_r - U_\infty \sin\theta \mathbf{e}_\theta, \tag{1.4}$$
$$P_e \longrightarrow P_\infty, \quad T_e \longrightarrow T_\infty,$$
$$r \longrightarrow 0\colon T_i \neq \infty.$$

Here, U_r and U_θ are the radial and shear component of mass velocity $\mathbf{U}_e$, $\mathbf{U}_\infty$ is the velocity of the plane-parallel flow of the fluid about the particle ($\mathbf{U}_\infty \parallel 0Z$), $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are the unit vectors of the spherical coordinate system, $U_\infty = |\mathbf{U}_\infty|$, σ_0 is the Stefan–Boltzmann constant, σ_1 is the integral blackness, and Δ is the Laplacian.

The basic parameters of the problem are coefficients ρ_e, μ_∞, and λ_e, as well as quantities R, T_∞, and U_∞, which remain unchanged during the motion of the spherical particle. From these parameters, one can constitute a dimensionless combination, the Reynolds number, $\mathrm{Re}_\infty = (\mu_\infty U_\infty R)/\rho_e \ll 1$. Let us nondimensionalize Eqs. (1.1)–(1.3) and boundary conditions (1.4) as follows: $\mathbf{V}_e = \mathbf{U}_e/U_\infty$, $t_k = T_k/T_\infty$, and $p_e = P_e/P_\infty$ ($P_\infty = (\mu_\infty U_\infty)/R$), where $k = e$ or i.

At $\varepsilon = \mathrm{Re}_\infty \ll 1$, the incoming flow can be considered as a disturbance; therefore, a solution to the equations of fluid dynamics and heat transfer should be sought in the form

$$\mathbf{V}_e = \mathbf{V}_e^{(0)} + \varepsilon \mathbf{V}_e^{(1)} + \ldots, \quad p_e = p_e^{(0)} + \varepsilon p_e^{(1)} + \ldots, \quad t = t^{(0)} + \varepsilon t^{(1)} + \ldots. \tag{1.5}$$

2. TEMPERATURE FIELDS AND THE DRIVING FORCE AND VELOCITY OF THE STEADY MOTION OF THE HYDROSOL PARTICLE

Substituting (1.5) into the set of Eqs. (1.1)–(1.3) and separating the variables, we arrive at expressions for the mass velocity components and temperature fields,

$$V_r(y, \theta) = \cos\theta(1 + A_1G_1(y) + A_2G_2(y)), \quad t_e(y, \theta) = t_e^{(0)}(y) + \varepsilon t_e^{(1)}(y, \theta); \tag{2.1}$$

$$V_\theta(y, \theta) = -\sin\theta(1 + A_1G_3(y) + A_2G_4(y)), \quad t_i(y, \theta) = t_i^{(0)}(y) + \varepsilon t_i^{(1)}(y, \theta), \tag{2.2}$$

where

$$G_1 = -\frac{1}{y^3}\sum_{n=0}^{\infty}\frac{\Delta_n^{(1)}}{(n+3)y^n},$$

$$G_2 = -\frac{1}{y}\sum_{n=0}^{\infty}\frac{\Delta_n^{(2)}}{(n+1)y^n}$$

$$-\frac{\alpha}{y^3}\sum_{n=0}^{\infty}\left[(n+3)\ln\frac{1}{y}-1\right]\frac{\Delta_n^{(1)}}{(n+3)^2 y^n},$$

$$G_3 = G_1 + \frac{y}{2}G_1^I, \quad G_4 = G_2 + \frac{y}{2}G_2^I, \tag{2.3}$$

$$t_e^{(0)}(y) = 1 + \frac{\gamma}{y}, \quad V = \frac{4}{3}\pi R^3,$$

$$t_i^{(0)}(y) = \left(B_0 + \frac{1+\omega}{4\pi R\lambda_{i\infty}T_\infty y}\int_V q_i dV + \int_V \frac{f_0}{y}dy - \frac{1}{y}\int_V f_0 dV\right)^{\frac{1}{1+\omega}}, \quad y = r/R,$$

$$t_i^{(1)}(y) = \frac{\cos\theta}{(1+\omega)t_{i0}^{\omega}}\left(By + \frac{(1+\omega)RJ}{3\lambda_{i\infty}T_\infty y^2} + \frac{1}{3}\left[y\int_1^y \frac{f_1}{y^2}dy - \frac{1}{y^2}\int_1^y f_1 dy\right]\right), \quad t_e^{(1)} = \frac{\Gamma}{y^2}\cos\theta,$$

$$f_n(y) = -\frac{R^2(1+\omega)}{\lambda_{i\infty}T_\infty}y^2\frac{2n+1}{2}\int_{-1}^{+1} q_i(r,\theta)P_n(x)dx,$$

$$J = \frac{1}{V}\int_V q_i z dV$$

is the dipole moment of the heat source density, $P_n(x)$ are Legendre polynomials, $x = \cos\theta$, and $z = r\cos\theta$.

In (2.3), G_k^I and G_k^{II} are the first and second derivatives of the corresponding functions with respect to y ($k = 1, 2$). Coefficients $\Delta_n^{(1)}$ $(n \geq 1)$ and $\Delta_n^{(2)}$ $(n \geq 3)$ are found with the recurrence relations

$$\Delta_n^{(1)} = -\frac{1}{n(n+5)} \times \sum_{k=1}^{\infty}[(n+4-k)(\alpha_k^{(1)}(n+5-k)-\alpha_k^{(2)})+\alpha_k^{(3)}]\gamma^k\Delta_{n-k}^{(1)},$$

$$\Delta_n^{(2)} = -\frac{1}{(n+3)(n-2)}\left[-6\alpha_n^{(4)}\gamma^{(n)} + \sum_{k=1}^{n}\{(n+2-k) \times [(n+2-k)\alpha_k^{(1)}-\alpha_k^{(2)}]+\alpha_k^{(3)}\}\gamma^k\Delta_{n-k}^{(2)} + \alpha\sum_{k=0}^{n}[(2n+5-2k)\alpha_k^{(1)}-\alpha_k^{(2)}]\gamma^k\Delta_{n-k-2}^{(1)}\right].$$

When calculating coefficients $\Delta_n^{(1)}$ and $\Delta_n^{(2)}$ by these formulas, one should take into account that $\Delta_0^{(1)} = -3$, $\Delta_0^{(2)} = -1$, $\Delta_2^{(2)} = 1$, $\alpha_0^{(1)} = \alpha_0^{(4)} = 1$, $\alpha_0^{(3)} = -4$, $\alpha_n^{(1)} = F_n$, $\alpha_n^{(2)} = (4-n)F_n + AF_{n-1}$, $\alpha_0^{(2)} = 4$, $\alpha_n^{(4)} = A^n/n!$, $\alpha_n^{(3)} = 2AF_{n-1} - 2(n+2)F_n$,

$$\Delta_1^{(2)} = -\frac{\gamma}{4}[6\alpha_1^{(4)} + 2(3\alpha_1^{(1)} - \alpha_1^{(2)}) + \alpha_1^{(3)}],$$

$$\alpha = \frac{\gamma}{15}\{-6\gamma\alpha_2^{(4)} + [3(4\alpha_1^{(1)} - \alpha_1^{(2)}) + \alpha_1^{(3)}]\Delta_1^{(2)} - [2(3\alpha_2^{(1)} - \alpha_2^{(12)}) + \alpha_2^{(3)}]\gamma\}.$$

The constants of integration appearing in (2.1) and (2.2) are found from the boundary conditions on the particle surface. Then, integrating the stress tensor over the particle surface, one can find total force **F** acting on the hydrosol particle, which is a linear combination of viscous force $\mathbf{F}_\mu$ and force $\mathbf{F}_q$,

$$\mathbf{F} = \mathbf{F}_\mu + \varepsilon\mathbf{F}_q, \tag{2.4}$$

where

$$\mathbf{F}_\mu = -6\pi R\mu_\infty U_\infty f_\mu \mathbf{n}_z, \quad \mathbf{F}_q = 6\pi R\mu_\infty f_q J\mathbf{n}_z,$$

$$f_\mu = \frac{2}{3}\frac{N_2}{N_1}\exp\{-A\gamma\},$$

$$f_q = K_{\rm ts}\frac{4}{3}\frac{\nu_{\rm es}}{t_{\rm es}}\frac{G_1}{N_1\delta\lambda_{\rm is}T_\infty}\exp\{-A\gamma\},$$

$$N_1|_{y=1} = G_1G_2^I - G_2G_1^I, \quad N_2|_{y=1} = -G_1^I,$$

$$t_{\rm es} = t_e^{(0)}|_{y=1},$$

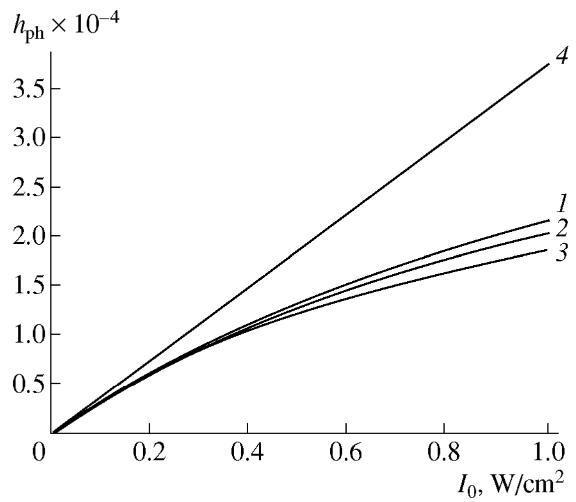
$$\delta = 1 + 2\frac{\lambda_{e\infty}}{\lambda_{\rm is}} + 4\frac{\sigma_0\sigma_1 R}{\lambda_{\rm is}}T_\infty^3 t_{\rm es}^3, \quad \lambda_{\rm is} = \lambda_{i\infty}t_{\rm es}^{\omega},$$

and $\mathbf{n}_z$ is the unit vector in the Z direction. Subscript "s" refers to physical quantities taken on the hydrosol particle surface at mean dimensionless temperature $t_{\rm es} = T_{\rm es}/T_\infty$. This temperature is given by

$$\frac{T_{\rm es}}{T_\infty} = 1 + \frac{1}{4\pi R\lambda_{e\infty}T_\infty}\int_V q_i(r,\theta)dV - \frac{\sigma_0\sigma_1 RT_\infty^3}{\lambda_{e\infty}}\left[\left(\frac{T_{\rm es}}{T_\infty}\right)^4 - 1\right]. \tag{2.5}$$

In (2.5), integration is over the entire volume of the particle.

With the condition $\lambda_e \ll \lambda_i$ satisfied, one can ignore the θ dependence of the dynamic viscosity in the particle–fluid system and put $\mu_e(t_e) \approx \mu_e(t_e^{(0)})$. Then, expres-



Function h_{ph} vs. incident radiation intensity I_0 for ω = (*1*) 0.5, (*2*) 0.7, and (*3*) 1.0. Curve *4* is constructed for small temperature drops.

sion (1) takes the form

$$\mu_e = \mu_\infty\left[1 + \sum_{n=1}^{\infty} F_n \frac{\gamma^n}{y^n}\right]\exp\left\{-A\frac{\gamma}{y}\right\}. \quad (2.6)$$

Formula (2.6) allows splitting the problem into thermal and hydrodynamic subproblems. Respective solutions are joined together using the boundary conditions on the surface of the hydrosol particle.

Equating total force **F** to zero yields an expression for the velocity of steady motion (steady-state velocity) of a solid spherical particle in a viscous nonisothermal fluid,

$$\mathbf{U}_q = -\varepsilon\frac{f_q}{f_\mu}J\mathbf{n}_z. \quad (2.7)$$

3. RESULTS AND DISCUSSION

Formulas (2.4) and (2.7) make it possible to estimate the effect of internal heat evolution on the driving force and velocity of steady motion of a hydrosol particle when the viscosity of the surrounding fluid is an exponential–power function of temperature.

It follows from these formulas that the sense and magnitude of $\mathbf{F}_q$ and $\mathbf{U}_q$ depend on the sense and magnitude of the dipole moment of the heat source density, as well as on the thermal conductivity of the particle. At $\lambda_i \longrightarrow \infty$ and a fixed value of the dipole moment of the heat source density, $\mathbf{F}_q$ and $\mathbf{U}_q$ tend to zero.

To estimate the effect of heat sources nonuniformly distributed over the volume of the hydrosol particle on its steady-state velocity, it is necessary to know their nature. Let heat sources arise when the particle acting as a black body absorbs electromagnetic radiation. In this case, the radiation is absorbed in a thin layer of thickness $\delta R \ll R$ adjacent to the area being heated and the heat source density within this layer is given by [10]

$$q_i(r,\theta) = \begin{cases} -\dfrac{I_0}{\delta R}, & \dfrac{\pi}{2} \le \theta \le \pi, \quad R - \delta R \le r \le R \\ 0, & 0 \le \theta \le \dfrac{\pi}{2}, \end{cases}$$

where I_0 is the incident radiation intensity.

Then,

$$\int_V q_i(r,\theta) z dV = -\frac{2}{3}\pi R^3 I_0, \quad \int_V q_i(r,\theta) dV = \pi R^2 I_0,$$

and we obtain expressions accounting for the effect of internal heat evolution on the driving force and velocity of steady motion of a solid spherical particle acting as an absolutely black body,

$$F_{ph} = -\varepsilon 6\pi R\mu_\infty K_{ts} f_{ph}, \quad U_{ph} = \varepsilon K_{ts} h_{ph},$$

where

$$f_{ph} = \frac{2}{3}\frac{\nu_{es}}{t_{es}}\frac{G_1}{N_1 \delta\lambda_{is} T_\infty}\exp\{-A\gamma\} I_0,$$

$$h_{ph} = \frac{\nu_{es}}{t_{es}}\frac{G_1}{N_2 \delta\lambda_{is} T_\infty} I_0.$$

The figure plots function h_{ph} against incident radiation intensity I_0 to illustrate the effect of internal heat evolution on phoretic velocity U_{ph} of the solid hydrosol particle. Numerical estimates were made for borated graphite particles suspended in water. The calculation parameters were T_∞ = 273 K, $\lambda_{i\infty}$ = 55 W/(m K), and three values of ω. Curve *4* in the figure was constructed for small relative temperature drops ($\gamma \longrightarrow 0$, $\omega = 0$) but for molecular transport coefficients taken at the particle surface mean temperature. Internal heat evolution is seen to significantly influence the steady-state velocity of the hydrosol particle.

CONCLUSIONS

We derived expressions for the force acting on a spherical hydrosol particle and for its steady-state velocity at an arbitrary temperature drop between the particle surface and the surrounding fluid away from the particle. The particle is heated by heat sources nonuniformly distributed in its volume. The results obtained in this work may be helpful in designing experimental setups imposing directional motion on hydrosol particles, estimating the sedimentation rate of hydrosol particles in channels, analyzing transport of hydrosol particles in the chemical reaction zone, etc. Quantitative investigation into this effect for solid particles seems to be a feasible task.

REFERENCES

1. S. Bretshneider, *Properties of Gases and Liquids: Calculation Techniques for Engineers* (Khimiya, Moscow, 1966) (translated from Polish).
2. V. B. Kutukov, E. R. Shchukin, and Yu. I. Yalamov, Zh. Tekh. Fiz. **46**, 626 (1976) [Sov. Phys. Tech. Phys. **21**, 361 (1976)].
3. E. R. Shchukin and N. V. Malai, Inzh.-Fiz. Zh. **54**, 628 (1988).
4. Yu. I. Yalamov, A. B. Poddoskin, and A. A. Yushkanov, Dokl. Akad. Nauk SSSR **254**, 1047 (1980) [Sov. Phys. Dokl. **25**, 734 (1980)].
5. K. J. Happel and H. Brenner, *Low Reynolds Number Hydrodynamics* (Prentice-Hall, Englewood Cliffs, 1965; Mir, Moscow, 1976).
6. S. A. Beresnev and L. B. Kochneva, Opt. Atmos. Okeana **16**, 134 (2003).
7. L. A. Dombrovskiĭ, Teplofiz. Vys. Temp. **42**, 961 (2004).
8. L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics*, Vol. 6: *Fluid Mechanics* (Nauka, Moscow, 1986; Pergamon, New York, 1987).
9. G. Z. Gershuni and E. M. Zhukhovitskii, *Convective Stability of Incompressible Fluids* (Nauka, Moscow, 1972; Springfield, Jerusalem, 1976).
10. C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles* (Wiley, New York, 1983; Mir, Moscow, 1986).

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