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An abrasion (undercut) slope diffusion model (ASDMO-1)

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with 3 figures and 1 table

Zusammenfassung. Der Aufsatz behandelt den funktionalen Zusammenhang zwischen Kliffunterschneidung, Profilversteilung, denudativer Kliffabtragung, Schuttakkumulation am Kliff-Fuß und Abfuhr dieses Schutts durch Wellen und Strömung. Dieser Zusammenhang, der mehrere Wechselwirkungen enthält, wird Schritt für Schritt mittels eines Gleichungssystems auf Grund vernünftiger geomorphologischer Annahmen bzw. geomorphologischer Erfahrung quantitativ beschrieben und als ein FORTRAN-Rechenprogramm (ASDMO-1) programmiert. Daraus wird dann für acht verschiedene Fälle die zeitliche Profilabfolge des rückwandernden Kliffs berechnet: mit konvexem und geradlinigen Anfangsprofil, mit Unterschneidung als negative Exponentialfunktion und als lineare Funktion des Schuttvolumens am Hangfuß sowie mit zwei verschiedenen Raten des küstenparallelen Materialtransports ($2^3 = 8$).

Summary. The first version of a system undercut slope diffusion model (ASDMO-1) with an account of a negative feedback has been developed. To realize a system model of slopes and abrasion processes interaction an algorithm is suggested. An algorithm and a programme (the FORTRAN language) are used for a computer of the ES-1020 type. Eight calculation variants are given.

Résumé. On développe la première version d'un modèle d'évolution des pentes par sapement (ASDMO-1) et on rapporte un feedback négatif. Un algorithm est suggéré pour réaliser un modèle de systèmes d'interactions entre les pentes et les processus d'abrasion. Un algorithm et un programme en langage Fortran sont utilisés sur un ordinateur de type ES-1020. Huit variantes de calculs sont données.

The first version of a slope retreat diffusion model system with a due regard to the negative feedback (ASDMO-1) on the basis of the undercut slope diffusion model has been constructed previously (TROFIMOV & MOSKOVKIN 1976; MOSKOVKIN & TROFIMOV 1980). Since a waste cover prevents a further cutting back of the base of the slope, it causes a mechanism of this feedback. The velocity v at which the slope foot is being cut back is supposed to be the function of the waste accumulating there: $v = F(w)$ where w is the waste volume. The following types of this function are simple and applicable:

(1) $\quad v = v_o \exp(-\alpha W), v = \beta (W_o - W)$

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where v_o, α, β = const; W_o is the maximum waste volume at which a process of undercutting at the slope foot stops.

The waste volume W is determined by a slope profile changing during the time t with a regard to a constant intensity of a waste transport (λ = const) by long shore currents and waves.

$$W = \int_O^\infty [y[x,o] - y(x,t)]\,dx;\; y(x,t) \equiv O \text{ when } O \leqslant x < s(t) \tag{2}$$

The modelling of the slope evolution process is carried out according to the undercut slope diffusion model (Trofimov & Moskovkin 1976)

$$\frac{\delta y}{\delta t} = K \frac{\delta^2 y}{\delta x^2},\; y(s(t), t) = O;\; s(t) \leqslant x < +\infty$$
$$y(x, o) = f(x) \Rightarrow y(x,t) \;(= \text{see } (4)),\; s(t) = \int_o^t b(t)\,d(t) \tag{3}$$

To realize the system model (1–3), which is the model of a slope and abrasion process interaction itself, the following algorithm is suggested (fig. 1). Consider v (t) = v_{01} to be constant during a small time span t_1 and solve the regional equation (3) with the simplified boundary condition y (v_{01} t, t) = O. The analytical solution of this equation (Trofimov & Moskovkin 1976) is as follows

$$y(x,t) = \frac{\exp\left(\frac{v^2}{4k} t - \frac{v}{2k} x\right)}{2\sqrt{\pi k t}} \int_o^\infty f(z) \exp\left(\frac{v\,z}{2\,k}\right)$$

$$\left\{\exp\left[-\frac{(z - x + vt)^2}{4\,k\,t}\right] - \exp\left[-\frac{(z + x - vt)^2}{4\,k\,t}\right]\right\} dz \tag{4}$$

Having obtained a slope profile shape at the time t_1, by this solution, then by the formular (2) $W_{01} = \int_o^\infty [y(x,o) - y(x,t_1)]\,dx - \lambda t_1$, the waste volume is calculated, taking into account its removal with the intensity λ. If W < O, the value W can be used in the formulas (1). Then, by one of the formulas (1), for instance, by the first $v_{12} = v_0 \exp[-\alpha W_{01}]$, one calculates a new velocity of cutting and considers this velocity to be constant during the intervals of time (t_1, t_2). By using the analytical solution (4) one solves the regional equation (3) with the boundary condition y (v_{12} t, t) = O and the initial one y (x, t_1) till the moment of the time t_2. Then the whole procedure is being repeated from the same beginning (fig. 1). The integral in the analytical solution is being calculated by Simpson's method and in the equation (2) it is being done by the Hermite-Gausse method, since Simpson's method in the latter case takes a long period of computing time (the polynome of Hermite of the 16th power is being considered: H_{16} (x)).

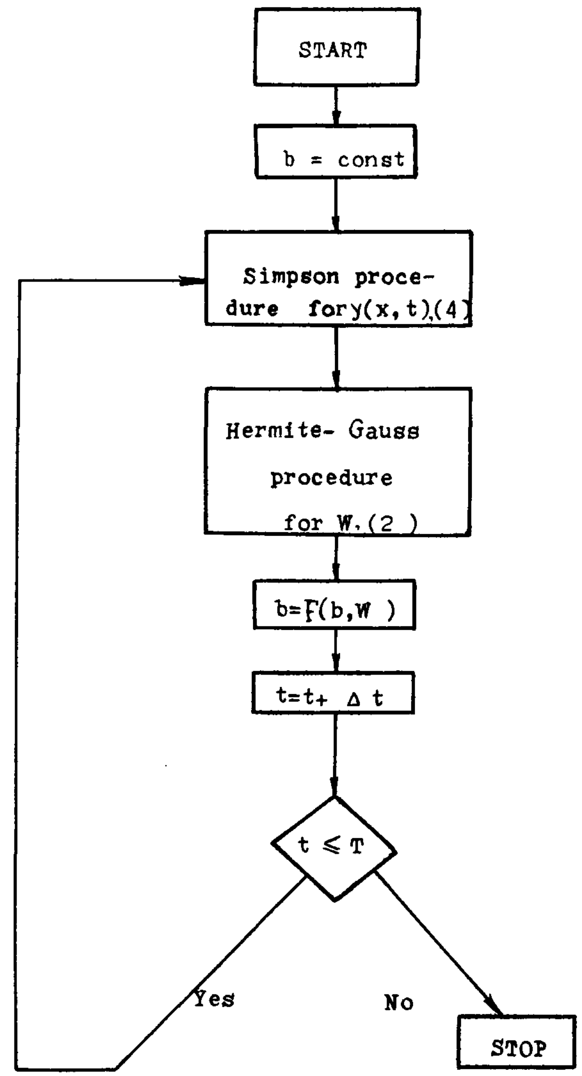

Fig. 1. Flow chart according to the ASDMO-1.

The algorithm and the program, recorded in FORTRAN, have been done for a computer of the ES-1020 type. Eight calculation variants have been calculated for the two initial slope profiles y (x, 0) = 40 [1 – exp (– x/100)], y (x, 0) = 0,4x; for the two types of the function v (W) (1) with v_0 = 1,0 m/yr; α = 1/100; β = (1/300) (m/yr)$^{-1}$; W_0 = 300 M^2 and for two waste transport intensities by long shore currents and waves (λ = 0; 10 m^2/yr).

The calculation (fig. 2) in all the variants were being carried out with the same denudation coefficients (k = 0,1 m^2/yr) and the same initial velocity (v_0 = 1,0 m/yr)

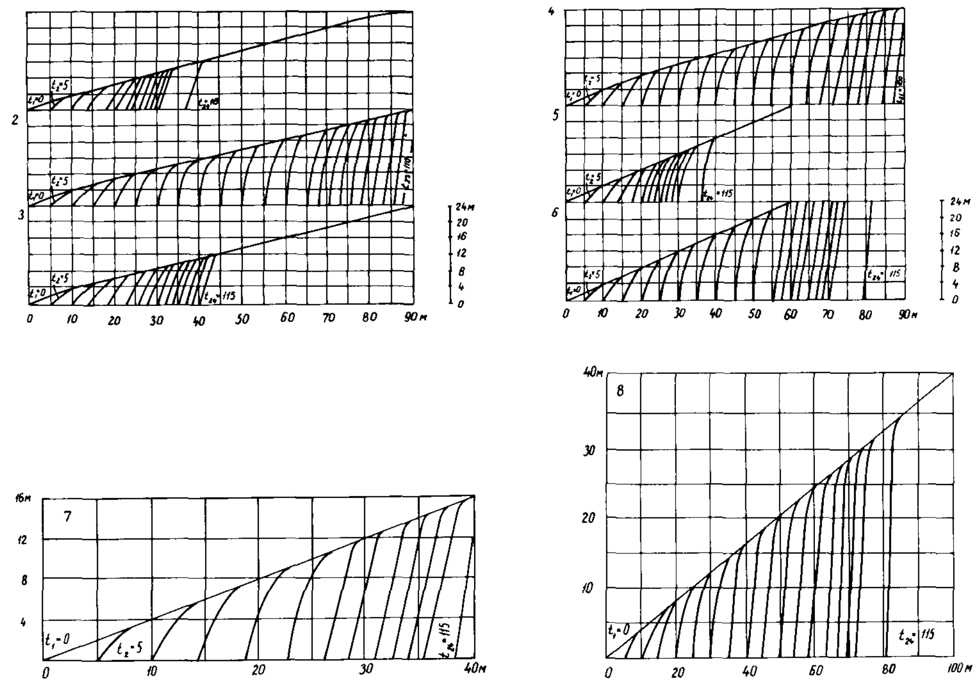

Fig. 2. Evolution of a slope according to the ASDMO-1: -variants 1–3, 4–6, 7, 8.

of basal cutting till the time t = T = 115 years, which approximately corresponded to the time needed for the stabilization of the cutting velocity (fig. 3).

The comparison of the first two variants, when the functions y (x, 0) = f (x) and v (W), which differ by the parameter λ only, are exponentially set, showed the slope to have retreated 52,69 m due to a larger waste transport intensity ($\lambda = 10\ m^2/yr$) in the second variant than in the first one. The constant cutting rate (v (W) = 1,0 m/yr) in the second variant was been observed up to 65 years.

Due to a large initially set rate of cutting, a steep slope (cliff) is rapidly formed. In 115 years it will reach 77,3 degrees in the first variant and in the second 82,8 degrees. In the 3rd – 4th variants, in contrast to the first two, a linear function of W is used for the rate of backcutting v instead of the exponential one. In the fourth variant, when λ = 10 m/yr, the slope retreats 52,77 m farther in 115 years than when a waste transport is absent (λ = 0, the thirtd variant). During 115 years the slope retreats 41,16–37, 65 = 3,51 (m) and 93,93–90. 34 = 3,59 (m), respectively, (table 1) farther than in the first two variants; this is caused by the more rapid expontial decrease of backcutting in the first two variants compared to the linear backcutting function in the second two.

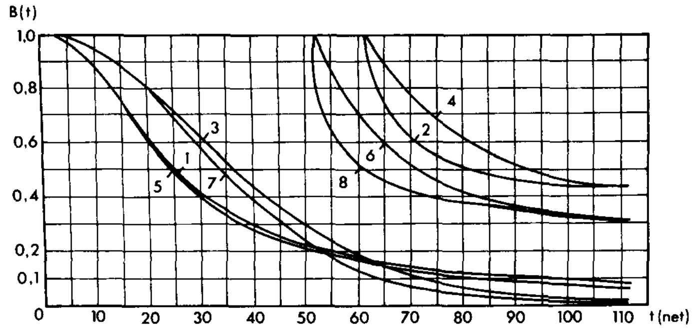

Fig. 3. Time dependence rate of a slope basement cutting for 8 calculation variants according to the ASDMO-1.

Table 1 The displacement of a slope base point according to the ASDMO-1 during a time for eight calculation variants.

t, yr	S (t), m							
	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
5	5,00	5,00	5,00	5,00	5,00	5,00	5,00	5,00
10	9,76	10,00	9,92	10,00	9,76	10,00	9,92	10,00
15	13,92	15,00	14,60	15,00	13,89	15,00	14,59	15,00
20	17,37	20,00	18,92	20,00	17,29	20,00	18,88	20,00
25	20,00	25,00	22,80	25,00	20,04	25,00	22,69	25,00
30	22,53	30,00	26,19	30,00	22,28	30,00	25,97	30,00
35	24,48	35,00	29,09	35,00	24,13	35,00	28,73	35,00
40	26,13	40,00	31,53	40,00	25,69	40,00	30,98	40,00
45	27,56	45,00	33,54	45,00	27,03	45,00	32,78	45,00
50	28,80	50,00	35,17	50,00	28,19	50,00	34,20	50,00
55	29,91	55,00	36,49	55,00	29,21	55,00	35,30	55,00
60	30,89	60,00	37,55	60,00	30,12	57,88	36,14	59,08
65	31,78	65,00	38,39	65,00	30,93	60,36	36,79	62,45
70	32,59	68,41	39,05	69,36	31,67	62,64	37,28	65,28
75	33,33	71,31	39,57	73,14	32,34	65,78	37,65	67,74
80	34,01	73,98	39,97	76,46	32,96	66,83	37,92	69,95
85	34,64	76,51	40,29	79,45	33,53	68,80	38,13	71,97
90	35,23	78,95	40,54	82,20	34,06	70,70	38,28	73,87
95	35,77	81,33	40,73	84,76	34,55	72,54	38,40	75,68
100	36,28	83,65	40,88	87,19	35,01	74,34	38,48	77,42
105	36,77	85,92	41,00	89,52	35,44	76,08	38,55	79,11
110	37,22	88,15	41,09	91,77	35,84	77,79	38,59	80,75
115	37,65	90,34	41,16	93,96	36,23	79,45	38,63	82,35

In the 5th–8th variant a linear initial slope profile was used; all other parameters and the function v (W) were used in the same order of variation as in the first four variants.

The 5th–6th variants were being considered with the exponential function v (W) and two values for λ (λ = 0; 10 m^2/yr). During 115 years in the 6th variant the slopes base had retreated 43,22 m farther than in the 5th variant. By the 115th year the inclination angles at the slope foot have correspondently reached 79 degrees for the 5th and 85 degrees for the 6th variant.

In the last two variants (7 an d8), the linear function v (W) was used; this caused a somewhat larger retreat of the slope base in comparison with the 5th and 6th ones: 82,35–79,45 = 2,9 (m) and 38,63–36,23 = 2,4 (m). By the 115th year the inclination angles had reached 80 and 85 degrees, respectively.

From the time v (t) all the odd-numbered variants (λ = 0) have led to convexo-concave curves of a cutting rate (fig. 3). The even variants (λ = 10 m^2/yr) have led to the constant function v (t) = 1,0 m/yr when t < 55 of 65 years and to exponentially damping functions when t > 55 or 65 years.

When the cutting rate decreases sufficiently, the formation of a gentler slope will begin. This is not observed in the given calculations due to the short duration of modelling. The increase of the steepening rate diminishes in time, i. e. there is a critical time when it reaches zero; after that, a process of slope flattening will begin.

The average calculation time is about two minutes with a computer of the ES-1020 type making 60 000 operations per second. The programme was compiled in such a way that besides slope heights it provides output in steps of $\Delta x = 3$ m, $\Delta t = 5$ years. The values of v (t) (fig. 3) and s (t) (table 1) are given as well. Besides forecasting slope formation problems, the model can be used in analyses of the disturbing effects of natural and manmade factors.

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