Publ. Univ. of Miskolc, Series A. Mining. Vol. 50. (1995) No. 2. pp. 133-141

OPTIMAL CONTROL OVER SHORE SYSTEM: CLIFF-BEACH IN CONDITIONS OF LOOSE-ROCKS

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SUMMARY

We consider the problems of the optimal control over shore system: cliff-beach, the operation which is described with the help of control dynamic system of the order second Pontragin's principle of maximum is used for the solution this problem. Some examples of the analytical solutions of this task for the concrete dependences of the rate and intensity of abrasion of the beach-forming in terms of material volume are considered.

In the paper problems of the optimal control over shore-system are considered: cliff-beach in conditions of linear and nonlinear law of the beach-forming material attrition.

The dynamics of the shore-system: cliff-beach is essentially defined by the rate of cliff retreat which under the conditions of loose-rocks may be approximated by the function $f(W) = \frac{b}{W}$ (b = const, m^3/yr) and the intensity of

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material attrition which in conditions of a linear law of attrition, has the form $\varphi(W) = KW$ (K = const, yr^{-1}) (Esin, 1980). Here W is the beach-forming material volume per unit length of the shore-line, m^2. Then based on equation of the beach-forming material balance under the conditions of control the following system of equations can be written

$$\frac{dW}{dt} = \frac{aHb}{W} - KW + u(t)$$

$$\frac{ds}{dt} = \frac{b}{W}, \quad |u(t)| \le \beta = const, \tag{1}$$

where a is a portion of the beach-forming material in the shoreforming bedrock; H is the cliff height, m; S is the distance of cliff retreat in the abrasion process, m; u is the intensity of material delivery ($u > 0$) or removal ($u < 0$), m^2/yr; β is the restriction put on the control factor; t is time, yr.

In some practical cases it is worthwhile to consider the task of the transference of dynamic system (1) from some initial state (W_o, 0) to the dynamical equilibriumal (stable) state (W_{st}, S_1) with a beforehand given value of cliff retreat $S = S_1$ at the least time (Moskovkin, 1985).

Here W_{st} is obtained from the first equation of the system (1) at $u = 0$, $\frac{dW}{dt} = 0$. Thus, we have

$$W_{st} = \sqrt{\frac{aHb}{K}}.$$

This problem of the optimal fast-actioning according to Pontragin's maximum principle (Boltyanski, 1969) is investigated on the basis of solution system of the equation (1) in the phasal plane (W, S) at $u = \beta$ and $u = -\beta$. Thus, synthesis of the optimal controls is obtained on the basis of solution of the equations

$$\frac{dW}{dS} = aH - \frac{KW^2}{b} + \frac{\beta W}{b}, \tag{2}$$

$$\frac{dW}{dS} = aH - \frac{KW^2}{b} - \frac{\beta W}{b}. \tag{3}$$

Considering equation (2), we obtain the class of phasal trajectories situated between the asymptotes

$$\overset{+}{W}_{1,2} = \frac{\beta}{2K} \pm \frac{\sqrt{A}}{2K}$$

in the form $S + C_1 = \frac{-2b}{\sqrt{A}} \, arth \left(\frac{\beta - 2KW}{\sqrt{A}}\right)$, (4)

where C_1 – constant of the integration; arth – inverse hyperbole tangent, the sign plus above W corresponds to the positive sign of the control, index 1 at W corresponds to sign plus in the form for $\overset{+}{W}_{1,2}$

$$A = \beta^2 + 4\,KabH$$

The class of phasal trajectories situated in the interval $W > \overset{+}{W}_1$ corresponds to the second class of the solutions of the equation (2)

$$S + C_2 = -\frac{2b}{\sqrt{A}} \, arcth \left(\frac{\beta - 2KW}{\sqrt{A}}\right), \tag{5}$$

where C_2 = const; $arcth$ – inverse hyperbole cotangent.

If the trajectory at $u = -\beta < 0$ is the switching line passing through the final point (W_{st}, S_1) then we obtain a concrete equation for it

$$S = S_1 + \frac{2b}{\sqrt{A}} \left[arcth \left(\frac{-2KW_{st} - \beta}{\sqrt{A}}\right) - arcth \left(\frac{-2KW - \beta}{\sqrt{A}}\right)\right]. \tag{6}$$

The equation of the phasal trajectory starting from a given initial point (W_o, 0) at W_o, < W_1, is obtained from the general solution (4)

$$S = -\frac{2b}{\sqrt{A}} \left[arcth \left(\frac{\beta - 2KW}{\sqrt{A}} - arcth\left(\beta - 2KW_o\right)\right)\right]. \tag{7}$$

An analogous trajectory can be obtained from the class of the solutions (5) at $W_o > \overset{+}{W}_1$. One of the coordinates of the switching point (W_{sw}) is obtained at the joint consideration of the equations (6) and (7)

$$W_{sw} = \frac{1}{K} \sqrt{\frac{1}{2} (\beta^2 + 2KabH + \beta \sqrt{A} \, th \, R)}$$

$$R = \frac{S_1 \sqrt{A}}{2b} + arcth \left[\frac{\sqrt{A} \, (K(W_o - W_{st}) - \beta}{\beta^2 + 2KabH + K\beta(W_{st} - W_o) - 2K^2 W_{st} W_o}\right]. \tag{8}$$

The second coordinate of the switching point (S_{sw}) is defined by substituting the form (8) in the form (6).

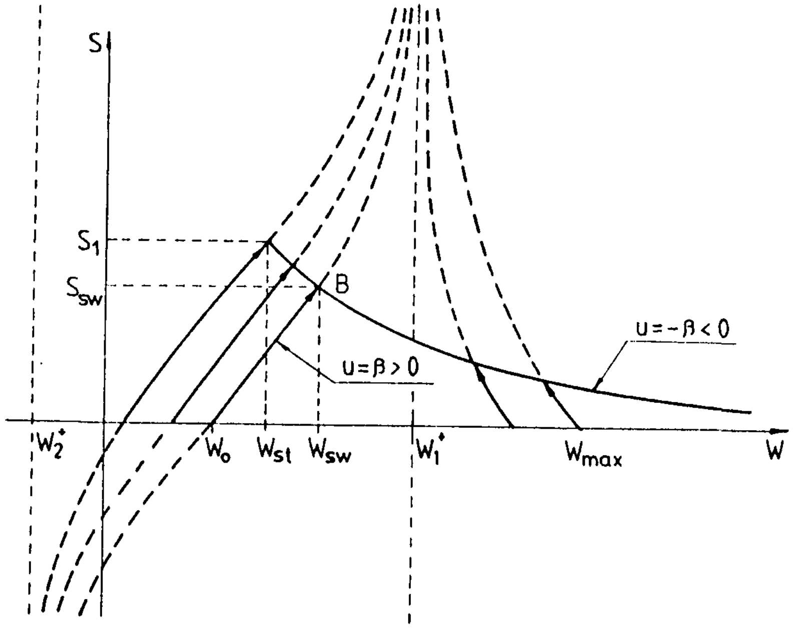

Figure 1.
The qualitative synthesis of the optimal controls for the dynamic system (1)

The general picture of the synthesis occurring here is shown in the *figure 1*. The time of the movement from the initial point (W_0, 0) at $W_0 < W_1$ to the switching point B with the coordinates (W_{sw}, S_{sw}) we find by integrating the first equation of the system of equations (1) at u(t) = β

$$t_{w_o, w_{sw}} = \frac{1}{2}K \ln \left| \frac{-KW_o^2 + \beta W_o + aHb}{-KW_{sw}^2 + \beta W_{sw} + aHb} \right| - \frac{\beta}{K\sqrt{A}} arth\left[\frac{\sqrt{A}\,(W_o - W_{sw})}{2abH + \beta\,(W_o + W_{sw} - 2K\,W_o\,W_{sw}}\right]. \quad (9)$$

The time of the movement from the switching point to the final point has the form:

$$t_{w_{sw}, w_{st}} = \frac{1}{2K} \ln \left| \frac{-KW_{sw}^2 - \beta W_{sw} + aHb}{-KW_{st}^2 - \beta W_{st} + aHb} \right| + \frac{\beta}{K\sqrt{A}} arcth\left[\frac{-2KW_{sw}\,W_{st} + \beta\,(W_{sw} + W_{st}) - 2abH)}{\sqrt{A}\,(W_{st} - W_{sw})}\right]. \quad (10)$$

The general optimal time is

$$T = t_{W_o, W_{sw}, W_{st}} + t_{W_{sw}, W_{st}}.$$

By analogy of the equations (6–10) the synthesis in the case can be calculated if the trajectory at u = β > 0 passing through a final point (W_{st}, S_1) may be to taken as the switching line (an alternative synthesis).

The results of the calculations on the first synthesis (egus. [6–10]) were written in the *table 1*, on the alternative synthesis – in the *table 2*. For each of the four versions there are two phasal trajectories issuing from one point (W, S) = (10, 0), moreover the trajectory with the negative control corresponds to the alternative synthesis. The initial constant parameters: a = 0,02, b = 40 m^3/yr (Esin, 1980, Loose loamy rocks in the region of the cape Burnas, Black Sea). The rest of variative parameters are shown in the tables. The calculated phasal portrait of the both syntheses for the second version of the tables 1 and 2 is shown at the *figure 2*.

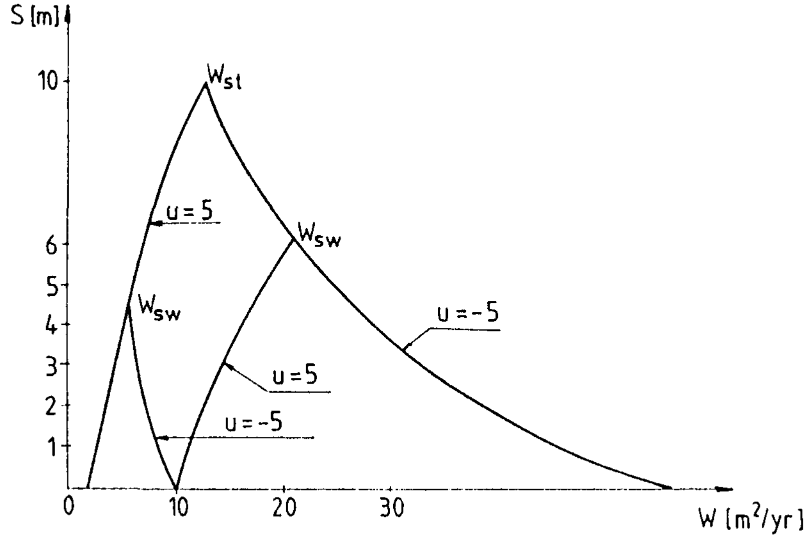

Figure 2.
Calculating synthesis of the optimal controls according to the second versions of the tables 1 and 2.

For a given realistic range of change of parametres of the model:
$5 \le \beta \le 10$ m^2/yr; $0{,}05 \le K \le 0{,}1$ yr^{-1}; $10 \le H \le 50$ m the general optimal time for the first synthesis variants in the interval from three to five years, for the second synthesis – in the interval from one to three years. Thus, the second synthesis (at the beginning is the removal, then the delivery) is more

optimal by the time, consequently and by the spends in the works bounding with delivery and removal of the material. This strategy of the control keeps and for the other values W_0, S_1 is optimal for a given class of the tasks independently of the initial parameters.

In case of nonlinear law of the beach-forming material attrition $\varphi(W) = \frac{cW}{\gamma + W}$ which is more suitable in case of large beachs material attrition we obtain the following equations for the calculation of synthesis of the optimal controls:

$$\frac{dW}{dS} = aH - \frac{CW^2}{b(\gamma + W)} + \frac{\beta W}{b} \tag{11}$$

$$\frac{dW}{dS} = aH - \frac{CW^2}{b(\gamma + W)} - \frac{\beta W}{b}. \tag{12}$$

The unique stable point is defined by the form

$$W_{st} = \frac{aHb}{2C} + \sqrt{\left(\frac{aHb}{2C}\right)^2 + \frac{aHb\gamma}{C}}. \tag{13}$$

By analogy we calculate the alternative synthesis, when the trajectory at $u = \beta > 0$ passing through the point (W_{st}, S_1) is taken as the swithcling line. The calculations obtained in case of the both syntheses are shown in *figure 3* and in *table 3*. The parameters a, b, S_1 were chosen as in the previous problem. The rest of the parameters are shown in the table 3, while W_{st} is calculated with the help of the formula (13).

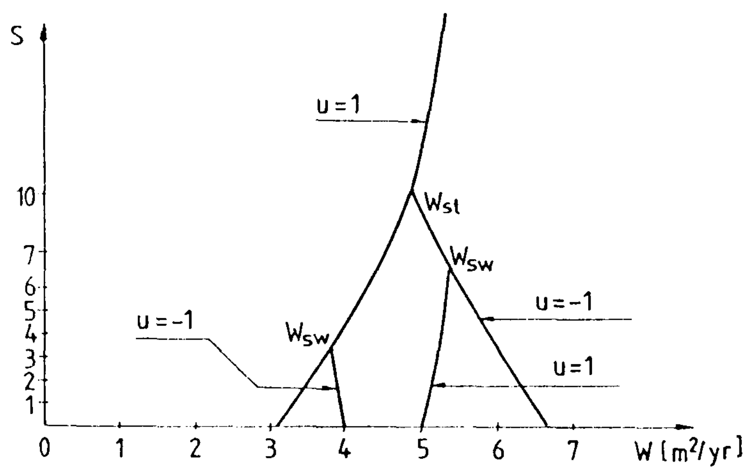

Figure 3.
Calculating synthesis of the optimal controls according to the table 3.

In the first version the trajectory is starting from the point (W_0, 0) = (4,0) with the negative sign of the control (u = −1 m^2/yr.; the removal of the material), in the second version the trajectory is issuing from the point (5,0) with reverse sign of the control (u = 1 m^2/yr.; the delivery of the material).

By quality the phasal portraites of the syntheses for considering our different functions $\varphi(W)$ almost does not differ. The coincidence occurres in the domain where the nonlinear function can be substituted by the linear function. So, for instance, the stable state W_{st} = 4,9 m^2 (the *table 3*) for the linear law of the attrition occurrs at K = 0,34 yr.$^{-1}$; and hence in this case the calculative syntheses are almost the same for both laws $\varphi(W)$.

The analogous tasks of the optimal control are possible for more complicated functions $f(W)$ (the abrasion rate), taking into account the optimization of different economics and recreative factors as well.

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