Прикладная математика & Физика, 2022, том 54, №1. С. 15–20.

УДК 517.9 DOI 10.52575/2687-0959-2022-54-1-15-20
MSC 35C07, 35C08, 35Q60

EXACT SOLUTIONS OF THE LAKSHMANAN – PORSEZIAN – DANIEL EQUATION

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(*Article submitted by a member of the editorial board Yu. P. Virchenko*)

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Received January, 31, 2022

Abstract. In this paper, the Lakshmanan – Porsezian – Daniel (LPD) equation is considered. This equation is integrable and admits Lax pair. The LPD equation is the generalization of the nonlinear Schrodinger (NLS) equation and described by Ablowitz-Kaup-Newell-Segur (AKNS) system. Using the sine-cosine method and the hyperbolic tangent method a variety of new exact solutions are obtained. These methods are effective tools for searching exact solutions of nonlinear partial differential equations in mathematical physics. The obtained solutions are found to be important for the explanation of some practical physical problems.

Key words: Lakshmanan – Porsezian – Daniel equation, AKNS, Lax pair, sine-cosine method, hyperbolic tangent method

Acknowledgements: This research is funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP09057947).

For citation: Shaikhova Gaukhar, Syzdykova Arailym, Kudaibergenov Gaziz. 2022. Exact solutions of the Lakshmanan – Porsezian – Daniel equation. Applied Mathematics & Physics. 54(1): 15–20. (in English)
DOI 10.52575/2687-0959-2022-54-1-15-20

ТОЧНЫЕ РЕШЕНИЯ УРАВНЕНИЯ ЛАКШМАНАНА – ПОРСЕЗИАНА – ДАНИЭЛЯ

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Аннотация. В данной работе рассмотрено уравнение Лакшманана – Порсезиана – Даниэля (ЛПД). Это уравнение интегрируемо и имеет пару Лакса. Уравнение ЛПД является обобщением нелинейного уравнения Шредингера и описывается системой Абловица – Каупа – Ньюэлла-Сегура (АКНС). В работе применены метод синус-косинуса и метод гиперболического тангенса, получены различные новые точные решения. Предложенные методы являются эффективными инструментами для поиска точных решений нелинейных дифференциальных уравнений в частных производных математической физики. Кроме того, полученные решения важны для объяснения некоторых практических задач физики.

Ключевые слова: уравнение Лакшманана – Порсезиана – Даниэля, АКНС, пара Лакса, метод синус-косинуса, метод гиперболического тангенса

Благодарности: Работа выполнена при финансовой поддержке Комитета науки Министерства образования и науки Республики Казахстан (номер гранта AP09057947).

Для цитирования: Шайхова Г. Н., Сыздыкова А. М., Кудайбергенов Г. 2022. Точные решения уравнения Лакшманана – Порсезиана – Даниэля. Прикладная математика & Физика. 54(1): 15–20. (in Russian)
DOI 10.52575/2687-0959-2022-54-1-15-20

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1. Introduction. Nonlinear partial differential equations are broadly used to model nonlinear processes in many areas of mathematical biology, physics, chemistry [18, 1]. As a result of interest in those problems, different analytical solution methods as Hirota's bilinear method [6, 7], Darboux transformation method [10, 21], sine-cosine method [19, 17], hyperbolic tangent method [11, 12] and so on were developed.

Studying the nonlinear excitations of the spin chains with competing bilinear and biquadratic interactions attracts is the main activity in mathematics and physics. For this reason, Lakshmanan, Porsezian, and Daniel had been studied the integrable properties of a classical one-dimensional isotropic biquadratic Heisenberg spin chain

(HSC) in its continuum limit by using a geometric method in Refs. [8, 15]. Researchers suggested the integrable Lakshmanan – Porsezian – Daniel (LPD) equation which has the higher-order terms (dispersions and nonlinear effects).

The LPD equation is given by [8, 15],

$$iq_t + q_{xx} + 2|q|^2 q + \gamma[q_{xxxx} + 8|q|^2 q_{xx} + 2q^2 q^*_{xx} + 4q|q_x|^2 + 6q^* q_x^2 + 6|q|^4 q] = 0, \tag{1}$$

where $q(x,t)$ is a complex valued function of the spatial coordinate x and the time t, γ is real constant, the subscripts denote the partial derivatives with respect to the variables x, t. The LPD equation is NLS type equation with higher-order nonlinear terms, such as fourth-order dispersion, second-order dispersion, cubic and quintic nonlinearities. It also describes the effect of higher-order molecular excitations that introduce quadruple–quadruple coefficients and is a candidate of integrable. Moreover, the LPD equation demonstrates many integrability properties like Painleve analysis, Lax pair representation, soliton solutions, and so on. More clearly the LPD equation describes the nonlinear effect in Refs. [8, 15, 5].

In the case $\gamma = 0$, (1) reduces into nonlinear Schrodinger equation

$$iq_t + q_{xx} + 2|q|^2 q = 0. \tag{2}$$

Linear eigenvalue problem for (1), which is obtained through the Ablowitz – Kaup – Newell-Segur (AKNS) system [2, 13, 14], is written as

$$\Psi_x = A\Psi, \tag{3}$$
$$\Psi_t = B\Psi, \tag{4}$$

with eigenfunctions as $\Psi = (\Psi_1, \Psi_2)^T$, and

$$A = -i\lambda\sigma_3 + M, \tag{5}$$

$$\begin{aligned} B = {} & [3i\gamma|q|^4 + i|q|^2 + i\gamma(q^* q_{xx} + qq *_{xx} - |q_x|^2) + 8i\gamma\lambda^4 + 2\lambda\gamma(qq_x^* - q_x q^*) - \\ & -2i\lambda^2(2\gamma|q|^2 + 1)]\sigma_3 - 8\gamma\lambda^3 M - 4i\gamma\lambda^2\sigma_3 M_x + 6i\gamma M^2 M_x \sigma_3 + \\ & +i\sigma_3 M_x + i\gamma\sigma_3 M_{xxx} + 2\lambda(M + \gamma M_{xx} - 2\gamma M^3), \end{aligned} \tag{6}$$

where

$$M = \begin{pmatrix} 0 & q \\ -q^* & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

λ is a parameter, so that

$$A_t - B_x + AB - BA = 0, \tag{7}$$

is equivalent to (1). Matrices A, B are Lax pair of (1). The compatibility condition (7) can be understood also as the zero curvature condition.

Optical solitons for the local (classical) LPD equation are found by modified simple equation method in [3], by the trial equation method [4], and by Riccati equation approach [16]. Dynamical behavior of solution in integrable nonlocal LPD equation is studied via Darboux transformation in Ref. [9]. Very recently inverse scattering transform has been applied in Ref. [20] where generalized nonlocal Lakshmanan – Porsezian – Daniel (LPD) equation is introduced, and its integrability as an infinite dimensional Hamilton dynamic system is established.

In this paper, we construct some new exact solutions for (1) by analytical methods. We study the LPD equation (1) by the sine-cosine method and the hyperbolic tangent method. Such methods have been widely applied for a wide variety of nonlinear partial differential equations to obtain different kind of solutions.

2. Sine–cosine Method. Due to in (1) $q(x,t)$ is complex function we apply the next transformation

$$q(x,t) = e^{i\alpha t} u(x), \tag{8}$$

to convert the LPD equation (1) into ordinary differential equation (ODE). After substitution (8) into (1) and making some algebraic manipulation we obtain ODE

$$-\alpha u + u'' + 2u^3 + \gamma[u'''' + 10u^2 u'' + 10u(u')^2 + 6u^5] = 0, \tag{9}$$

where α, γ are real constants and a prime mark denotes a derivative by independent variable x. In the next subsection, (9) can be solved by applying the sine-cosine method in variable x.

2.1 The Sine Solution. According to method the sine solution of the (9) can be found by transformation

$$u(x) = \lambda \sin^{\beta}(\mu x), \tag{10}$$

where parameters λ, μ and β will be determined, and μ is wave number. We use (10) and its derivatives

$$\begin{aligned} u^{'}(x) &= \lambda\beta\mu \sin^{\beta-1}(\mu x)\cos(\mu x), & (11)\\ u^{''}(x) &= -\mu^2\beta^2\lambda \sin^{\beta}(\mu x) + \mu^2\lambda\beta(\beta-1)\sin^{\beta-2}(\mu x), & (12)\\ u^{''''}(x) &= \mu^4\beta^4\lambda \sin^{\beta}(\mu x) - 2\mu^4\lambda\beta(\beta-1)(\beta^2-2\beta+2)\sin^{\beta-2}(\mu x) + \\ &+ \mu^4\lambda\beta(\beta-1)(\beta-2)(\beta-3)\sin^{\beta-4}(\mu x). & (13)\end{aligned}$$

After substitution of Eqs. (10)-(13) into (9) we obtain

$$\begin{aligned} -\alpha\lambda\sin^{\beta}(\mu x) - \mu^2\beta^2\lambda\sin^{\beta}(\mu x) + \mu^2\lambda\beta(\beta-1)\sin^{\beta-2}(\mu x) + \\ +2\lambda^3\sin^{3\beta}(\mu x) + \gamma\mu^4\beta^4\lambda\sin^{\beta}(\mu x) - 2\gamma\mu^4\lambda\beta(\beta-1)(\beta^2-2\beta+2)\sin^{\beta-2}(\mu x) + \\ +\gamma\mu^4\lambda\beta(\beta-1)(\beta-2)(\beta-3)\sin^{\beta-4}(\mu x) - 20\lambda^3\gamma\mu^2\beta^2\sin^{3\beta}(\mu x) + \\ +10\mu^2\lambda^3\gamma\beta(\beta-1)\sin^{3\beta-2}(\mu x) + 10\lambda^3\gamma\mu^2\beta^2\sin^{3\beta-2}(\mu x) + 6\lambda^5\gamma\sin^{5\beta}(\mu x) = 0. \end{aligned} \tag{14}$$

Using the balance method, by equating the exponents of sin^k from (14) we find β:

$$\beta - 4 = 5\beta \quad \Rightarrow \quad \beta = -1. \tag{15}$$

Substitute (15) in (14) we obtain

$$\begin{aligned} -\alpha\lambda\sin^{-1}(\mu x) - \mu^2\lambda\sin^{-1}(\mu x) + 2\mu^2\lambda\sin^{-3}(\mu x) + 2\lambda^3\sin^{-3}(\mu x) + \\ +\gamma\mu^4\lambda\sin^{-1}(\mu x) - 20\gamma\mu^4\lambda\sin^{-3}(\mu x) + 24\gamma\mu^4\lambda\sin^{-5}(\mu x) - 20\lambda^3\gamma\mu^2\sin^{-3}(\mu x) + \\ +20\mu^2\lambda^3\gamma\sin^{-5}(\mu x) + 10\lambda^3\gamma\mu^2\sin^{-5}(\mu x) + 6\lambda^5\gamma\sin^{-5}(\mu x) = 0. \end{aligned} \tag{16}$$

From (16) we have the next system

$$\begin{aligned} \sin^{-1}(\mu x) &: -\alpha\lambda - \mu^2\lambda + \gamma\mu^4\lambda = 0, & (17)\\ \sin^{-3}(\mu x) &: 2\mu^2\lambda + 2\lambda^3 - 20\gamma\mu^4\lambda - 20\lambda^3\gamma\mu^2 = 0, & (18)\\ \sin^{-5}(\mu x) &: 24\gamma\mu^4\lambda + 20\mu^2\lambda^3\gamma + 10\lambda^3\gamma\mu^2 + 6\lambda^5\gamma = 0. & (19)\end{aligned}$$

Solving the last system yields

$$\alpha = -\frac{9}{100\gamma}, \quad \mu = \pm\sqrt{\frac{1}{10\gamma}}, \quad \lambda = \pm\sqrt{-\frac{1}{10\gamma}}. \tag{20}$$

Substituting (20) into (10) and then obtained expression into (8) we obtain the solitary wave solution and the periodic solution

$$\begin{aligned} q_1(x,t) &= \pm\sqrt{-\frac{1}{10\gamma}}csch(\sqrt{\frac{1}{10\gamma}}x)e^{-\frac{9i}{100\gamma}t}, \quad \gamma < 0, \\ q_1(x,t) &= \pm\sqrt{-\frac{1}{10\gamma}}csc(\sqrt{\frac{1}{10\gamma}}x)e^{-\frac{9i}{100\gamma}t}, \quad \gamma > 0. \end{aligned} \tag{21}$$

2.2 The Cosine Solution. According to method the cosine solution of the (9) can be found by transformation

$$u(x) = \lambda \cos^{\beta}(\mu x), \tag{22}$$

where parameters λ, μ and β will be determined, and μ is wave number. We use (22) and its derivatives

$$\begin{aligned} u^{'}(x) &= -\lambda\beta\mu \cos^{\beta-1}(\mu x)\sin(\mu x), & (23)\\ u^{''}(x) &= -\mu^2\beta^2\lambda \cos^{\beta}(\mu x) + \mu^2\lambda\beta(\beta-1)\cos^{\beta-2}(\mu x), & (24)\\ u^{''''}(x) &= \mu^4\beta^4\lambda cos^{\beta}(\mu x) - 2\mu^4\lambda\beta(\beta-1)(\beta^2-2\beta+2)\cos^{\beta-2}(\mu x) + \\ &+ \mu^4\lambda\beta(\beta-1)(\beta-2)(\beta-3)\cos^{\beta-4}(\mu x). & (25)\end{aligned}$$

Substituting Eqs. (22)-(25) into (9) we obtain

$$-\alpha\lambda\cos^{\beta}(\mu x)-\mu^2\beta^2\lambda\cos^{\beta}(\mu x)+\mu^2\lambda\beta(\beta-1)\cos^{\beta-2}(\mu x)+$$
$$+2\lambda^3\cos^{3\beta}(\mu x)+\gamma\mu^4\beta^4\lambda\cos^{\beta}(\mu x)-2\gamma\mu^4\lambda\beta(\beta-1)(\beta^2-2\beta+2)\cos^{\beta-2}(\mu x)+$$
$$+\gamma\mu^4\lambda\beta(\beta-1)(\beta-2)(\beta-3)\cos^{\beta-4}(\mu x)-20\lambda^3\gamma\mu^2\beta^2\cos^{3\beta}(\mu x)+$$
$$+10\mu^2\lambda^3\gamma\beta(\beta-1)\cos^{3\beta-2}(\mu x)+10\lambda^3\gamma\mu^2\beta^2\cos^{3\beta-2}(\mu x)+6\lambda^5\gamma\cos^{5\beta}(\mu x)=0. \tag{26}$$

From (26) by using the balance method we find β:

$$\beta-4=5\beta \quad\Rightarrow\quad \beta=-1. \tag{27}$$

After substitution (27) in (26) we obtain

$$-\alpha\lambda\cos^{-1}(\mu x)-\mu^2\lambda\cos^{-1}(\mu x)+2\mu^2\lambda\cos^{-3}(\mu x)+2\lambda^3\cos^{-3}(\mu x)+$$
$$+\gamma\mu^4\lambda\cos^{-1}(\mu x)-20\gamma\mu^4\lambda\cos^{-3}(\mu x)+24\gamma\mu^4\lambda\cos^{-5}(\mu x)-20\lambda^3\gamma\mu^2\cos^{-3}(\mu x)+$$
$$+20\mu^2\lambda^3\gamma\cos^{-5}(\mu x)+10\lambda^3\gamma\mu^2\cos^{-5}(\mu x)+6\lambda^5\gamma\cos^{-5}(\mu x)=0. \tag{28}$$

From (28) we have the next system of equations

$$\cos^{-1}(\mu x)\ :\ -\alpha\lambda-\mu^2\lambda+\gamma\mu^4\lambda=0, \tag{29}$$
$$\cos^{-3}(\mu x)\ :\ 2\mu^2\lambda+2\lambda^3-20\gamma\mu^4\lambda-20\lambda^3\gamma\mu^2=0, \tag{30}$$
$$\cos^{-5}(\mu x)\ :\ 24\gamma\mu^4\lambda+20\mu^2\lambda^3\gamma+10\lambda^3\gamma\mu^2+6\lambda^5\gamma=0. \tag{31}$$

Solving the last system yields

$$\alpha=-\frac{9}{100\gamma},\quad \mu=\pm\sqrt{\frac{1}{10\gamma}},\quad \lambda=\pm\sqrt{-\frac{1}{10\gamma}}. \tag{32}$$

Substituting (32) into (22) and then obtained expression into (8) we obtain the solitary wave solution and the periodic solution

$$q_2(x,t)=\pm\sqrt{-\frac{1}{10\gamma}}sech(\sqrt{\frac{1}{10\gamma}}x)e^{-\frac{9i}{100\gamma}t},\quad \gamma<0,$$
$$q_2(x,t)=\pm\sqrt{-\frac{1}{10\gamma}}sec(\sqrt{\frac{1}{10\gamma}}x)e^{-\frac{9i}{100\gamma}t},\quad \gamma>0. \tag{33}$$

3. The Hyperbolic Tangent Method. In this section, we use the hyperbolic tangent method as presented by Malfliet [11, 12] to ODE (9)

$$-\alpha u+u''+2u^3+\gamma[u''''+10u^2u''+10u(u')^2+6u^5]=0. \tag{34}$$

According to method, we apply the following series expansion,

$$u(x)=S(Y)=\sum_{k=0}^{M}a_kY^k, \tag{35}$$

where $Y=\tanh(\mu x)$ and M is a positive integer, in most cases, that will be determined. To determine the parameter M, we usually balance the linear terms of highest-order derivative in the resulting equation with the highest-order nonlinear terms. For our (34), balancing the nonlinear term u^5, which has the exponent $5M$, with the highest order derivative u'''', which has the exponent $M+4$, yields $5M=M+4$ that gives $M=1$. Then, the hyperbolic tangent method allows us to use the substitution

$$u(x)=a_0+a_1Y, \tag{36}$$

where

$$Y=\tanh(\mu x),$$

and derivatives by method are

$$\frac{du}{dx} = \mu(1-Y^2)\frac{du}{dY}, \tag{37}$$

$$\frac{d^2u}{dx^2} = -2\mu^2 Y(1-Y^2)\frac{du}{dY} + \mu^2(1-Y^2)^2\frac{d^2u}{dY^2}, \tag{38}$$

$$\frac{d^4u}{dx^4} = -8\mu^4 Y(1-Y^2)(3Y^2-1)\frac{du}{dY} + 4\mu^4(1-Y^2)^2(9Y^2-2)\frac{d^2u}{dY^2} - -12\mu^4 Y(1-Y^2)^3\frac{d^3u}{dY^3} + \mu^4(1-Y^2)^4\frac{d^4u}{dY^4}, \tag{39}$$

and so on. After substitution Eqs. (36)-(39) into (34), collecting the coefficients of Y^n, and solving the resulting system with the aid of Maple, we find the following result:

$$a_0 = 0, \quad a_1 = \pm\sqrt{-\frac{1}{5\gamma}}, \quad \mu = \pm\frac{\sqrt{5}}{10\sqrt{\gamma}}, \quad \alpha = -\frac{4}{25\gamma}. \tag{40}$$

By substituting Eqs. (40) into (36), and then the obtained expression into (8), we can obtain the periodic solution and the solitary wave solution for the LPD equation (1) in the following forms

$$q_3(x,t) = \pm e^{-\frac{4i}{25\gamma}t}\sqrt{-\frac{1}{5\gamma}}tan(\frac{\sqrt{5}}{10\sqrt{\gamma}}x), \quad \gamma < 0, \tag{41}$$

$$q_3(x,t) = \pm e^{-\frac{4i}{25\gamma}t}\sqrt{-\frac{1}{5\gamma}}tanh(\frac{\sqrt{5}}{10\sqrt{\gamma}}x), \quad \gamma > 0. \tag{42}$$

4. Conclusion. In this work, the Lakshmanan – Porsezian – Daniel equation was studied by the sine-cosine method and the hyperbolic tangent method. We obtained the periodic solutions and the solitary wave solutions. These methods can also be performed to other nonlinear partial differential equations in mathematical physics.

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Конфликт интересов: о потенциальном конфликте интересов не сообщалось.
Conflict of interest: no potential conflict of interest related to this article was reported.

Получена 25.01.2022

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