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THE MULTIPOLE METHOD FOR CERTAIN ELLIPTIC EQUATION WITH DISCONTINUOUS COEFFICIENT

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Abstract. A new analytic–numerical method has been developed for solving BVPs in 3D domains with cones of arbitrary base for certain elliptic equation with piecewise constant coefficient. The solution is obtained by the use of special basic functions — the Multipoles, which are costructed in an explicit form. The method supplies high accurate evaluation of the solution, its derivatives, singularity exponents and intensity factors near the geometrical singularities — edges and the corner vertex.

Keywords: boundary value problem, domains with cones, multipole method, singularity exponents, intensity factors.

1 Introduction

We consider boundary value problems (BVPs) for certain elliptic equation with piecewise constant coefficient in domains with cones of arbitrary base (particularly, with polyhedral corners), when the surface of discontinuity of the coefficient (interface surface) is a conical one passing through the vertex of the initial cone. An equivalent statement, which is called a transmission problem, consists in solving the Laplace equation with so called transmission conditions on the interface surface [1].

Solutions of such BVPs have singularities at vertices of the cones [2]-[8]. Development of effective methods for solving these BVPs, including high accuracy computation of the singularity exponents, became a challenging issue [1], [9]-[13].

In this work we present a new effective analytic–numerical method for high–accuracy computation of these singularities at cones of arbitrary base (in particular, for polyhedral corners), when conical interface surface also has an arbitrary base. This method represents a generalization of the Multipole method, previously developed in [14]-[16] for solving a certain class of 2D and 3D elliptic BVPs in domains of complex shape with geometric singularities of different kinds. For the case of the Laplace equation, the Multipole method in domains with cones has been developed in [17]-[19].

The principle underlying our method consists in using a system of basic functions that conform to the structure of the solution near the conical surfaces of the boundary and interface. We call these functions Multipoles due to their similarity to ordinary multipoles, known in the theory of potential [20]. Such systems possess good approximation properties. Most important is the fact that these basic functions can be expressed in explicit analytic form in terms of special functions.

By virtue of these features the method proves most effective for precise computation of exponents at the cone singularity.

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2 Statement of the problem

2.1 Domains $\mathcal{K}$ and Ω

Let (r, θ, φ) be spherical co-ordinates of a point x in space $\mathbb{R}^3$. Denote by

$$\mathbb{S}^2 := \{r = 1,\ \theta \in [0, \pi],\ \varphi \in [0, 2\pi)\}$$

the unit sphere and by $\mathbb{B}^3$ the unit ball in $\mathbb{R}^3$. Points $\theta = 0$ and $\theta = \pi$ on $\mathbb{S}^2$ are called the North Pole $\mathcal{P}_N$ and South Pole $\mathcal{P}_S$, respectively.

Consider two disjoint Lipschitz piecewise smooth contours $\mathcal{L}$ and $\mathcal{L}_{in}$ on the sphere $\mathbb{S}^2$, each divides $\mathbb{S}^2$ into two domains, one of which contains $\mathcal{P}_S$ and another $\mathcal{P}_N$. The domain containing $\mathcal{P}_N$ and bounded by $\mathcal{L}$ (by $\mathcal{L}_{in}$) is denoted by $\mathcal{S}$ (by $\mathcal{S}_{in}$). Assume that $\mathcal{L}_{in} \subset \mathcal{S}$ and denote $\mathcal{S}_{ex} = \mathcal{S} \setminus \overline{\mathcal{S}}_{in}$; observe that $\partial\mathcal{S}_{ex} = \mathcal{L} \cup \mathcal{L}_{in}$.

The domain $\mathcal{K} \subset \mathbb{R}^3$ defined by the formula $\mathcal{K} := \{r \in (0, \infty),\ (\theta,\ \varphi) \in \mathcal{S}\}$ is an (infinite) cone with base $\mathcal{S}$, its boundary being the conical surface $\partial\mathcal{K} = \{r \in (0, \infty),\ (\theta,\ \varphi) \in \mathcal{L}\}$. The domains $\mathcal{K}_{in}, \mathcal{K}_{ex}$ and their boundaries $\partial\mathcal{K}_{in},\ \partial\mathcal{K}_{ex}$ are defined in a similar way, with the vertex $\{0\}$ shared by both cones, $\overline{\mathcal{K}} = \overline{\mathcal{K}}_{in} \cup \overline{\mathcal{K}}_{ex}$, conical surface $\partial\mathcal{K}_{in}$ contained in $\mathcal{K} \cup \{0\}$, and $\partial\mathcal{K}_{ex} = \partial\mathcal{K} \cup \partial\mathcal{K}_{in}$.

Consider an important instance of cone $\mathcal{K}$ when it presents a trihedral corner with its three faces being plane angles with common vertex $\{0\}$ and with values of the angles being equal to $\pi\alpha$, where $\alpha \in (0, 2/3]$. Denote by $\mathcal{K}^\alpha$ this trihedral corner, by $\mathcal{S}^\alpha$ its base, and by $\mathcal{L}^\alpha$ the contour of this base. In this instance, the equation of contour $\mathcal{L}^\alpha$ can be written in the form

$$\mathcal{L}^\alpha = \{(\theta, \varphi) : \theta = \theta(\varphi),\ \varphi \in [0, 2\pi)\},\ \theta(\varphi) = \begin{cases} \mathrm{T}(\varphi + \frac{2\pi}{3});\ \varphi \in [0,\ \frac{2\pi}{3}], \\ \mathrm{T}(\varphi);\quad \varphi \in [\frac{2\pi}{3},\ \frac{4\pi}{3}], \\ \mathrm{T}(\varphi - \frac{2\pi}{3});\ \varphi \in [\frac{4\pi}{3},\ 2\pi], \end{cases} \tag{2.1}$$

with function $\mathrm{T}(\varphi)$ given by the formula

$$\mathrm{T}(\varphi) = \arccos\left[\cos\varphi / \sqrt{c + \cos^2\varphi}\right] \tag{2.2}$$

that involves parameter $c = (1 - \cos\pi\alpha)\left(2 + 4\cos\pi\alpha\right)^{-1}$. It worth to be mentioned that value $\pi\beta$ of dihedral angle between faces of $\mathcal{K}^\alpha$ are related to the quantity $\pi\alpha$ by the formula $\cos\pi\alpha = \cos\pi\beta / (1 - \cos\pi\beta)$.

The transmission BVP is being solved in a domain $\Omega \subset \mathcal{K}$ homeomorphic to $\mathbb{B}^3$ with Lipschitz piecewise smooth boundary $\partial\Omega$. By definition, boundary $\partial\Omega$ consists of the two disjoint parts: $\partial\Omega = \gamma \cup \Gamma$, where γ is a closure of a simply-connected domain on the conical surface $\partial\mathcal{K}$ with its vertex $\{0\}$ being an interior point of γ, and $\Gamma \subset \mathcal{K}$ is a simply-connected domain on a certain piecewise smooth surface. Note that $\mathcal{K}$ is an extension of Ω through Γ. Assume that $\partial\mathcal{K}_{in}$ divides Ω into two subdomains Ω_{in} and Ω_{ex}. Define $\gamma_{in} = \partial\mathcal{K}_{in} \cap \overline{\Omega}$ and observe that $\gamma_{in} = \partial\Omega_{in} \cap \partial\Omega_{ex}$. Note that γ_{in} is the interface surface within domain Ω where the transmission conditions are to be set.

Let the surface Γ be divided by a Lipschitz piecewise smooth curve or contour into two domains: $\mathcal{D}$ and $\mathcal{N}$; the latter correspond to the boundary conditions (the Dirichlet or Neumann type) to be set on the corresponding parts of Γ.

2.2 The formulation of the transmission BVP with mixed Dirichlet — Neumann boundary conditions

For a function ψ defined on Ω, denote by ψ_{in} (by ψ_{ex}) its restriction to Ω_{in} (to Ω_{ex}). Consider the following transmission BVP for the Laplace equation in the domain Ω:

$$\Delta \psi_{in} = 0 \quad \text{in } \Omega_{in}, \qquad \Delta \psi_{ex} = 0 \quad \text{in } \Omega_{ex}, \tag{2.3}$$

with the transmission conditions on the interface surface

$$\psi_{in}\big|_{\gamma_{in}} = \psi_{ex}\big|_{\gamma_{in}}, \qquad \varkappa_{in}\, \partial_\nu\, \psi_{in}\big|_{\gamma_{in}} = \varkappa_{ex}\, \partial_\nu\, \psi_{ex}\big|_{\gamma_{in}}, \tag{2.4}$$

where $\varkappa_{in}$ and $\varkappa_{ex}$ are prescribed positive constants, ∂_ν being a normal derivative, and with mixed Dirichlet — Neumann type conditions

$$\psi\,\big|_{\gamma} = 0, \qquad \psi\,\big|_{\mathcal{D}} = h_{\mathcal{D}}, \qquad \partial_\nu \psi\,\big|_{\mathcal{N}} = h_{\mathcal{N}} \tag{2.5}$$

on the boundary $\partial\Omega = \gamma \cup \Gamma$.

We shall use the notation $h(x)$ defined by equalities

$$h(x) = h_{\mathcal{D}}(x), \quad x \in \mathcal{D}; \qquad h(x) = h_{\mathcal{N}}(x), \quad x \in \mathcal{N}, \tag{2.6}$$

and notation $\varkappa$ defined by the formula

$$\varkappa = \varkappa_{in}, \quad x \in \Omega_{in}; \qquad \varkappa = \varkappa_{ex}, \quad x \in \Omega_{ex}. \tag{2.7}$$

Transmission problem (2.3)–(2.5) can be rewritten in a generalized statement [6]-[8], [21]-[24]. In order to do it, Sobolev spaces are introduced, following [23]-[26].

Denote by $\overset{\circ}{W}{}_2^1(\Omega, \gamma)$ a subspace of $W_2^1(\Omega)$ consisting of functions having zero trace on γ. Similarly, define the space $\overset{\circ}{W}{}_2^1(\Omega, \gamma \cup \mathcal{D})$ as a subspace of $W_2^1(\Omega)$ consisting of functions with zero trace on $\gamma \cup \mathcal{D}$.

Let A be a subdomain of boundary $\partial\Omega$, and let a be a subdomain of A. Denote by $\overset{\circ}{W}{}_2^{1/2}(A, a)$ a subspace of the Sobolev — Slobodetskii space $W_2^{1/2}(A)$ consisting of functions vanishing a.e. on a. Only the particular cases of the latter spaces $\overset{\circ}{W}{}_2^{1/2}(\gamma \cup \mathcal{D}, \gamma)$ and $\overset{\circ}{W}{}_2^{1/2}(\partial\Omega, \gamma \cup \mathcal{D})$ are to be employed below. The so called negative space $\overset{\circ}{W}{}_2^{-1/2}(\partial\Omega, \gamma \cup \mathcal{D})$ is defined as a conjugate space to $\overset{\circ}{W}{}_2^{1/2}(\partial\Omega, \gamma \cup \mathcal{D})$.

The boundary data $h_{\mathcal{D}}$ and $h_{\mathcal{N}}$ in conditions (2.5) are required to belong to the spaces

$$h_{\mathcal{D}} \in \overset{\circ}{W}{}_2^{1/2}(\gamma \cup \mathcal{D}, \gamma), \qquad h_{\mathcal{N}} \in \overset{\circ}{W}{}_2^{-1/2}(\partial\Omega, \gamma \cup \mathcal{D}). \tag{2.8}$$

A generalized solution of BVP (2.3)–(2.5) is understood to be a function $\psi \in \overset{\circ}{W}{}_2^1(\Omega, \gamma)$ satisfying boundary condition $\psi\big|_{\mathcal{D}} = h_{\mathcal{D}}$ and the integral identity

$$\int_\Omega \varkappa\, (\nabla\psi,\, \nabla\eta)\, dx = \int_{\mathcal{N}} h_{\mathcal{N}}\, \eta\, ds$$

for all test-functions $\eta \in \overset{\circ}{W}{}_2^1(\Omega, \gamma \cup \mathcal{D})$, where the notation $(\,.\,,\,.\,)$ stands for the inner product in Euclidean space $\mathbb{R}^3$, and $\varkappa$ is defined by (2.7).

Solvability of the formulated BVP is guaranteed by the following

Theorem 1. *For any $h_{\mathcal{D}}$ and $h_{\mathcal{N}}$ satisfying (2.8) there exists a unique generalized solution $\psi \in \overset{\circ}{W}{}_2^1(\Omega, \gamma)$ of the problem (2.3)–(2.5).*

It is clear that Theorem 1 admits a standard proof which reduces to the Riesz representation theorem and follows well-known patterns (see e.g. [24]). Outside the boundary's singularities, regularity of the generalized solution of (2.3)–(2.5) is covered by the standard theory of elliptic BVPs [3], [6], [8], [21], [23]. Namely, the generalized solution is infinitely differentiable at any interior point $x \in \Omega \setminus \gamma_{in}$ as well as at any interior point of γ. At γ_{in}, the generalized solution is differentiable one-sidedly, i.e. on either side of γ_{in}, as many times as allows the smoothness of γ_{in}. Omitting the details, we just mention that regularity of the generalized solution at boundary points $x \in \mathcal{D}$ and $x \in \mathcal{N}$ depends on the smoothness of boundary surface Γ and boundary data $h_{\mathcal{D}}$, $h_{\mathcal{N}}$.

3 Construction of the system of basic functions (the Multipoles)

3.1 Reduction to a spectral problem for the Beltrami — Laplace operator with transmission conditions

Our goal consists in constructing a system of functions Ψ_k (the Multipoles) that possess good approximation properties and conform to the structure of the solution near the conical surfaces, which contain singularities (the vertex and edges). The basic functions are defined on the whole cone domain $\mathcal{K}$; their restrictions to $\mathcal{K}_{in}$ and $\mathcal{K}_{ex}$ are denoted by $\Psi_{k,in}$ and $\Psi_{k,ex}$, respectively. The desired properties of the basic functions require the following conditions to be met: 1) functions Ψ_k identically satisfy the Laplace equation in $\mathcal{K}$ with transmission conditions (2.4) on $\partial\mathcal{K}_{in}$; 2) they identically meet the homogeneous Dirichlet condition $\Psi_k = 0$ on $\partial\mathcal{K}$; 3) they constitute an orthogonal basis in $L_2(\mathbb{S})$.

The Multipoles are represented in the form

$$\Psi_k(r, \theta, \varphi) = r^{\mu}\, U(\mu; \theta, \varphi), \qquad \mu = \mu(k)\,; \qquad k = 1, 2, \ldots\,; \tag{3.1}$$

restrictions of $U(\mu; \theta, \varphi)$ to $\mathbb{S}_{in}$ and to $\mathbb{S}_{ex}$ are denoted by U_{in} and by U_{ex}, respectively.

Thus $U(\mu(k); \theta, \varphi) = U_k$ are eigenfunctions with eigenvalues $\mu(k)$ for the Laplace — Beltrami operator in domain $\mathbb{S}$ on the unit sphere

$$\frac{1}{\sin\theta}\,\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial U}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\,\frac{\partial^2 U}{\partial^2\varphi} + \mu\,(\mu + 1)\,U = 0 \qquad \text{in } \mathbb{S} \setminus \mathcal{L}_{in}\,, \tag{3.2}$$

with the transmission conditions on interface line $\mathcal{L}_{in}$, induced by (2.4), and with homogeneous Dirichlet condition on $\mathcal{L}$:

$$U_{in}\big|_{\mathcal{L}_{in}} = U_{ex}\big|_{\mathcal{L}_{in}}, \qquad \varkappa_{in}\,\partial_\nu\, U_{in}\big|_{\mathcal{L}_{in}} = \varkappa_{ex}\,\partial_\nu\, U_{ex}\big|_{\mathcal{L}_{in}}, \qquad U\big|_{\mathcal{L}} = 0\,. \tag{3.3}$$

Denote by $\overset{\circ}{W}{}_2^1(\mathbb{S})$ a subspace of $W_2^1(\mathbb{S})$ consisting of functions having zero trace on $\mathcal{L}$. A generalized solution of BVP (3.2), (3.3) is understood to be a function $U \in \overset{\circ}{W}{}_2^1(\mathbb{S})$ satisfying the integral identity

$$\int_{\mathbb{S}} \varkappa\,(\,\nabla_{\mathbb{S}} U,\ \nabla_{\mathbb{S}} V\,)\,ds = \mu\,(\mu + 1)\int_{\mathbb{S}} U\,V\,ds \qquad \forall\, V \in \overset{\circ}{W}{}_2^1(\mathbb{S}), \tag{3.4}$$

where $\nabla_{\mathbb{S}}$ stands for a tangential component to $\mathbb{S}$ of the gradient ∇. Note that an inner product

$$[U, V]_{\mathbb{S}} = \int_{\mathbb{S}} \varkappa (\nabla_{\mathbb{S}} U, \nabla_{\mathbb{S}} V)\, ds$$

induces an equivalent norm on $\overset{\circ}{W}{}_2^1(\mathbb{S})$.

Theorem 2. *For a spectral problem (3.4), there exists a denumerable set of generalized solutions $U = U_k \in \overset{\circ}{W}{}_2^1(\mathbb{S})$, $\mu = \mu(k)$, $k = 1, 2, \ldots$. The eigenvalues $\mu(k)$ have no finite limit points, and $\mu(k) \to \infty$ as $k \to \infty$. To each eigenvalue $\mu(k)$ there corresponds at most a finite number of generalized eigenfunctions $U_k \in \overset{\circ}{W}{}_2^1(\mathbb{S})$. The eigenfunctions $\{U_k\}$ form a basis in $L_2(\mathbb{S})$ and $W_2^1(\mathbb{S})$, which is orthonormal in $L_2(\mathbb{S})$ and orthogonal with respect to the inner product $[\,.\,,\,.\,]_{\mathbb{S}}$.*

It is clear that Theorem 2 admits a standard proof following the pattern of [21].

Remark 1. *In accordance with Theorem 2, all eigenvalues $\mu(k)$, $k = 1, 2, \ldots$ can be enumerated in order of their nondecresing; each multiple eigenvalue should be counted accordind to its multiplicity. Such renumbering establishes a one-to-one correspondence between eigenvalues $\mu(k)$ and eigenfunctions U_k.*

3.2 Solution of the spectral problem

In what follows we restrict ourselves to the case of contours $\mathfrak{L}$, $\mathfrak{L}_{in}$ being star–like on $\mathbb{S}^2$ with respect to North Pole, when $\mathfrak{L}$ can be represented in the form

$$\mathfrak{L} = \{(\theta, \varphi) : \theta = \theta(\varphi),\ \theta(\varphi) \in C(-\infty, +\infty),\ \theta(\varphi) = \theta(\varphi + 2\pi)\}$$

and $\mathfrak{L}_{in}$ can be represented in a similar form.

The eigenfunctions of the problem (3.2), (3.3) are constructed using two systems of complex–valued functions: $\{u^m(\mu;\, \theta,\, \varphi)\}_{m=0}^{\infty}$ and $\{v^m(\mu;\, \theta,\, \varphi)\}_{m=0}^{\infty}$ defined by the formulas:

$$u^m(\mu;\, \theta,\, \varphi) = \mathrm{P}_{\mu}^{m}(\cos\theta)\, e^{im\varphi}, \qquad v^m(\mu;\, \theta,\, \varphi) = \mathrm{P}_{\mu}^{m}(-\cos\theta)\, e^{im\varphi}, \tag{3.5}$$

where $\mathrm{P}_{\mu}^{m}(t)$ are associated Legendre functions on the cut [27]. For short, in complicated expressions we reduce the relations (3.5) to $u^m(\mu)$, $v^m(\mu)$.

Note that if $\mathcal{K}$ is a circular cone, i.e. $\mathfrak{L}$ is a circumference $\{\theta = \theta_0 = \text{const}\}$, then $\mathrm{Re}\, u^m(\mu; \theta, \varphi)$ and $\mathrm{Im}\, u^m(\mu; \theta, \varphi)$ are eigenfunctions of the problem (3.2), (3.3) with $\mu = \mu_n^m$ being the root of number $n\,(n = 1, 2, \ldots)$ of the equation $\mathrm{P}_{\mu}^{m}(\cos\theta_0) = 0$. Taking this fact into account we rename and renumber eigenvalues $\mu(k)$ as μ_n^m and eigenfunctions $U\big(\mu(k);\, \theta, \varphi\big)$ as $U_n^{m\,+}(\theta, \varphi)$ and $U_n^{m\,-}(\theta, \varphi)$.

Denoting restrictions of $U_n^{m\,\pm}(\theta, \varphi)$ to $\mathbb{S}_{in}$ and $\mathbb{S}_{ex}$ by $U_{n,\,in}^{m\,\pm}(\theta, \varphi)$ and $U_{n,\,ex}^{m\,\pm}(\theta, \varphi)$, respectively, let represent the desired eigenfunctions in the form of expansions in terms of functions (3.5):

$$U_{n,\,in}^{m\,\pm} = \mathrm{Re} \sum\nolimits_{l=0}^{\infty} A_n^{m,\,l\,\pm}\, u^{m+l}(\mu), \qquad A_n^{m,\,0\,+} = 1, \qquad A_n^{m,\,0\,-} = i, \tag{3.6}$$

$$U_{n,\,ex}^{m\,\pm} = \mathrm{Re} \sum\nolimits_{l=0}^{\infty} \Big\{ B_n^{m,\,l\,\pm}\, u^{m+l}(\mu) + C_n^{m,\,l\,\pm}\, v^{m+l}(\mu) \Big\}, \quad \mu = \mu_n^m. \tag{3.7}$$

Observe that functions (3.6), (3.7) with any coefficients identically satisfy the equation (3.2). Unknown eigenvalues μ_n^m and coefficients $A_n^{m,l\pm}$, $B_n^{m,l\pm}$, $C_n^{m,l\pm}$ in representations (3.6), (3.7) should be found from relations (3.3), which unite the transmission conditions on interface line $\mathfrak{L}_{in}$ and boundary condition on outer contour $\mathfrak{L}$.

We shall make it in the following way. Functions $U_n^{m\pm}(\theta,\varphi)$ are sought as a limit

$$U_n^{m\pm}(\theta,\varphi) = \lim_{M\to\infty} U_n^{m\pm}(M;\theta,\varphi)$$

of consequent approximations $U_n^{m\pm}(M;\theta,\varphi)$ written in the form of finite sums (3.6), (3.7) with coefficients depending on the length M of approximation, i.e.

$$U_{n,in}^{m\pm}(M;\theta,\varphi) = \mathrm{Re}\sum_{l=0}^{M} A_n^{m,l\pm}(M)u^{m+l}, \quad A_n^{m,0+}(M) = 1,\ A_n^{m,0-}(M) = i, \tag{3.8}$$

$$U_{n,ex}^{m\pm}(M;\theta,\varphi) = \mathrm{Re}\sum_{l=0}^{M}\left\{B_n^{m,l\pm}(M)\,u^{m+l} + C_n^{m,l\pm}(M)\,v^{m+l}\right\}. \tag{3.9}$$

Coefficients $A_n^{m,l\pm}(M)$, $B_n^{m,l\pm}(M)$, $C_n^{m,l\pm}(M)$ and approximate eigenvalues $\mu_n^{m\pm}(M)$ are determined by substituting $U_n^{m\pm}(M)$ into the transmission and boundary conditions (3.3) and by projecting the result onto the system of trigonometric functions $\exp(iq\varphi)$:

$$\left(U_{n,ex}^{m\pm}(M),\ \exp(iq\varphi)\right)_{\mathfrak{L}} = 0, \quad \left(U_{n,ex}^{m\pm}(M) - U_{n,in}^{m\pm}(M),\ \exp(iq\varphi)\right)_{\mathfrak{L}_{in}} = 0, \tag{3.10}$$

$$\left(\varkappa_{ex}\,\partial U_{n,ex}^{m\pm}(M)/\partial\nu - \varkappa_{in}\,\partial U_{n,in}^{m\pm}(M)/\partial\nu,\ \exp(iq\varphi)\right)_{\mathfrak{L}_{in}} = 0, \tag{3.11}$$

where $q = m,\dots,m+M$, and $(f_1,f_2)_{\mathfrak{L}}$ or $(f_1,f_2)_{\mathfrak{L}_{in}}$ is the inner product in $L_2(\mathfrak{L})$ or in $L_2(\mathfrak{L}_{in})$. Substituting representations (3.8), (3.9) into relations (3.10), (3.11) we obtain a system of linear equations with respect to coefficients $A_n^{m,l\pm}(M)$, $B_n^{m,l\pm}(M)$, $C_n^{m,l\pm}(M)$:

$$\mathfrak{D}^m(\mu)\,\mathfrak{Z} = 0, \tag{3.12}$$

where

$$\mathfrak{Z} = \left[A_n^{m,0\pm}(M), B_n^{m,0\pm}(M), C_n^{m,0\pm}(M), \dots, A_n^{m,M\pm}(M), B_n^{m,M\pm}(M), C_n^{m,M\pm}(M)\right]^T$$

is a vector of the coefficients. Elements of matrix $\mathfrak{D}^m(\mu)$ of system (3.12) are expressed as integrals over contours $\mathfrak{L}$ or $\mathfrak{L}_{in}$ of products of functions (3.5) or there normal derivatives on $\mathfrak{L}_{in}$; so, these elements depend only on number m and parameter μ.

In order to find a nontrivial solutions of homogeneous system (3.12), we equate the determinant of its matrix to zero, and in the issue we obtain the relation $\det\mathfrak{D}^m(\mu) = 0$, which should be considered as a transcidental equation with respect to μ. So, eigenvalue $\mu_n^m(M)$ is a root of number n $(n = 1, 2, \dots)$ of this equation.

The performed numerical experiments showed that the approximate eigenvalues and eigenfunctions converge to the exact ones. Namely, there hold the relations: 1) for any compact $E \subset \mathfrak{S}$ it holds

$$\lim_{M\to\infty}\left[\max_{(\theta,\varphi)\in E}\left|U_n^{m\pm}(M;\theta,\varphi) - U_n^{m\pm}(\theta,\varphi)\right|\right] = 0;$$

2) for all coefficients in (3.8), (3.9) it holds

$$A_n^{m,l\pm}(M) \to A_n^{m,l\pm}, \quad B_n^{m,l\pm}(M) \to B_n^{m,l\pm}, \quad C_n^{m,l\pm}(M) \to C_n^{m,l\pm} \text{ as } M\to\infty;$$

3) for all eigenvalues it holds $\mu_n^m(M) \to \mu_n^m$ as $M \to \infty$.

3.3 Computation of integrals of frequently oscillating functions

One of important computational problems arising in the described algorithm is calculation of elements of matrix $\mathfrak{D}^m(\mu)$ of system (3.12); those elements are expressed in the form of integrals over contours $\mathfrak{L}$ or $\mathfrak{L}_{in}$ of the following type:

$$\int \mathrm{P}_\mu^a\big(\cos\theta(\varphi)\big)\,\exp(\,ib\varphi\,)\,d\varphi\,, \qquad (3.13)$$

where $\theta(\varphi)$ is an equation of the contour; a and b are natural numbers, possibly very large. So, (3.13) are integrals with frequently oscillating integrand; effective computation of those integrals is a well–known challenging problem. A special analytic–numerical method has been developed for computation of such integrals. This method represents integrals (3.13) as exponentially convergent series involving integrals $\int_0^{\pi/2}\cos^\alpha t\cos(\beta t)\,dt$ and related ones, which we have computed explicitly via special functions, for whose computation high effective methods have been developed [28]. Particularly,

$$\int_0^{\pi/2}\cos^\alpha t\cos(\beta t)\,dt\;=\pi\,(1\,+\,\alpha)\,2^{-1-\alpha}\left[\mathrm{B}\left(1+\frac{\alpha+\beta}{2},\,1+\frac{\alpha-\beta}{2}\right)\right]^{-1},$$

where $\mathrm{B}(x,\,y)$ is Beta–function [27].

3.4 Numerical results

Note, that input data for the spectral transmission BVP (3.2), (3.3) consist, at first, of geometric data, determined by outer contour $\mathfrak{L}$ and interface line $\mathfrak{L}_{in}$, and, at second, of mechanical quantity $\kappa := \varkappa_{in}/\varkappa_{ex}$.

The method of solving this problem described in Sect. 3.2 has been realized for two types of geometric data. For type I contour $\mathfrak{L}$ is $\mathfrak{L}^\alpha$ turned to the angle δ $\mathfrak{L} = \{(\theta,\,\varphi) : (\theta,\,\varphi-\delta)\in\mathfrak{L}^\alpha\}$, and $\mathfrak{L}_{in} = \mathfrak{L}^{\alpha_{in}}$, $\alpha_{in} > \alpha$. Remind, that contour $\mathfrak{L}^\alpha$ is defined by (2.1), (2.2).

For type II contour $\mathfrak{L} = \mathfrak{L}^\alpha$, and interface line $\mathfrak{L}_{in} = \{(\theta,\,\varphi) : \theta = \theta_0,\,\forall\varphi\}$.

The dependence of eigenvalues μ_1^0 and μ_2^0 on κ is given on Fig. 1a and Fig. 1b, respectively, for type I of geometric data and for two variants of parameters: 1) $\alpha = 5/12$, $\delta = 1/6$, $\alpha_{in} = 7/12$, 2) $\alpha = 1/3$, $\delta = 1/6$, $\alpha_{in} = 1/2$. The graphs demonstrate considerable dependence of eigenvalues on κ.

For type II of geometric data with parameters $\alpha = 5/12$, $\theta_0 = 2/3$, $\kappa = 10$ the space views of the first $U_1^{0\,+}$ and the second $U_2^{0\,+}$ eigenfunctions with eigenvalues $\mu_1^0 = 0.090288$ and $\mu_2^0 = 1.453002$ are displayed on Fig. 2 and Fig. 3, respectively. The space views represent $2D$ graphs of the eigenfunctions, in which coordinates (θ,φ) are transformed by stereographic projection of the sphere $\mathbb{S}^2$ onto a plane (x_1,x_2), tangential to $\mathbb{S}^2$ at the North Pole.

4 The Solution of the Transmission BVP in Domain Ω

4.1 The Multipoles Ψ_k

In accordance with Theorem 2, all eigenvalues $\mu_n^m\,(M)$ can be enumerated as $\mu(k)$, $k = 1, 2, \ldots$, in order of their nondecreasing; each multiple eigenvalue should be counted according to its multiplicity. Thus, there arises respective numeration of the eigenfunctions $U_n^{m\,\pm}(\theta,\,\varphi)$ as

$U(\mu(k);\,\theta,\,\varphi)$ and, as a consequence, respective numeration of the multipoles $\Psi_k(r,\,\theta,\,\varphi)$; this manner of their numeration had already appeared in (3.1).

If our cone $\mathcal{K}$ is in fact a polyhedral angle, then a suitable representation for the Multipoles can be given. In order to formulate this representation let introduce a new system of spherical co-ordinates (r, Θ, Φ) related to an edge of the polyhedral angle.

Let us select a particular edge and define new Cartesian co-ordinates X, Y, Z with their origin at the vertex $\{0\}$ of the polyhedral angle disposed in such a way that the selected edge lies on axis Z, axis X lies on a face (or its extension), and axis Y is perpendicular to this face and is directed inside domain $\mathcal{K}$. Radial co-ordinate in the new system (r, Θ, Φ) coincides with the above one, and angle co-ordinates are defined by the standard formulas $\Phi = \arctan(Y/X)$, $\Theta = \arccos(Z/r)$. Denote the relation between old and new spherical co-ordinates by $\theta = \theta(\Theta,\,\Phi)$, $\varphi = \varphi(\Theta,\,\Phi)$. Then the desired representation for $U_k\,(\theta,\,\varphi) = V_k\,(\Theta,\,\Phi)$ has the form

$$V_k\,(\Theta,\,\Phi) \;=\; D_k^m\, \mathrm{P}_{\mu_k}^{-m/\beta}(\cos\Theta)\sin\frac{m\,\Phi}{\beta}\,. \tag{4.1}$$

Coefficients D_k^m in (4.1) can be computed as an integral over any curve $\{\Theta = \Theta_0 = \mathrm{const}\} \subset \mathcal{S}_{ex}$:

$$D_k^m \,= 2\left[\pi\,\beta\,\mathrm{P}_{\mu_k}^{-m/\beta}(\cos\Theta_0)\right]^{-1}\int_0^{\pi\,\beta} U_k\,(\theta,\,\varphi)\,\sin\frac{m\,\Phi}{\beta}\,d\Phi,$$

where $\theta = \theta\,(\Theta_0,\,\Phi)$, $\varphi = \varphi\,(\Theta_0,\,\Phi)$.

4.2 The method of solving BVP

Now we turn to the transmission BVP (2.3)–(2.5) in domain Ω with cones of arbitrary base as described in Sect. 2. Note that $\partial\Omega$ and γ_{in} may have at most a finite number of edges and conical points. Since the boundary $\partial\Omega$ is Lipschitz, a Sobolev space $\mathring{W}_2^1(\mathcal{D})$ is defined habitually as a subspace of $W_2^1(\mathcal{D})$ consisting of functions having zero trace on $\partial\mathcal{D}$. Obviously, the space $\mathring{W}_2^1(\mathcal{D})$ is a Hilbert space with the inner product

$$[u,\,v;\;\mathring{W}_2^1(\mathcal{D})] = \int_{\mathcal{D}} u\,v\,ds \,+\, \int_{\mathcal{D}} (\nabla_\Gamma\, u,\,\nabla_\Gamma\, v)\,ds\,,$$

where ∇_Γ stands for a tangential component to Γ of the gradient ∇. In the following theorem, notation $W_2^{3/2}(\Omega)$ stands for the Sobolev — Slobodetskii space with the norm, where standard notations are used (see, e.g. [21], [23], [26]),

$$\|\psi; W_2^{3/2}(\Omega)\|^2 = \|\psi; W_2^{1}(\Omega)\|^2 + \sum_{|\alpha|=1}\int_{\Omega\times\Omega}\frac{|\mathcal{D}_x^\alpha\,\psi(x) - \mathcal{D}_y^\alpha\,\psi(y)|^2}{|x\,-\,y|^4}\,dx\,dy\,.$$

Theorem 3. *Let $h_{\mathcal{D}} \in \mathring{W}_2^1(\mathcal{D})$ and $h_{\mathcal{N}} \in L_2(\mathcal{N})$. Then the generalized solution $\psi \in \mathring{W}_2^1(\Omega,\,\gamma)$ in Theorem 1 belongs to $W_2^{3/2}(\Omega)$, and*

$$\|\psi;\;W_2^{3/2}\,(\Omega)\|\;\le\;C\left(\,\|h_{\mathcal{D}};\;\mathring{W}_2^1(\mathcal{D})\|\,+\,\|h_{\mathcal{N}};\;L_2(\mathcal{N})\|\,\right)$$

with constant $C > 0$ depending only on Ω.

Due to the embedding $W_2^{3/2}(\Omega)$ into $W_2^1(\partial\Omega)$ the trace on $\partial\Omega$ of the generalized solution $\psi \in W_2^{3/2}(\Omega)$ in Theorem 3 belongs to $W_2^1(\partial\Omega)$. Denote by $H(\Gamma)$ a space of all generalized solutions $\psi \in \mathring{W}_2^1(\Omega,\gamma) \cap W_2^{3/2}(\Omega)$ in Theorem 3 with boundary data $h_{\mathcal{D}} \in \mathring{W}_2^1(\mathcal{D})$ and $h_{\mathcal{N}} \in L_2(\mathcal{N})$. Clearly, Theorem 3 implies that $H(\Gamma)$ is a Hilbert space with the inner product

$$[u, v]_H = \int_{\mathcal{D}} uv ds + \int_{\mathcal{D}} (\nabla_\Gamma u, \nabla_\Gamma v) ds + \int_{\mathcal{N}} \partial_\nu u \partial_\nu v\, ds.$$

For $u \in W_2^{3/2}(\Omega)$, existence of the trace $\nabla_\Gamma u \in L_2(\mathcal{D})$ is guaranteed by the embedding of $W_2^{3/2}(\Omega)$ into $W_2^1(\partial\Omega)$. Notice that existence of trace $\partial_\nu u \in L_2(\mathcal{N})$ is guaranteed only for the functions $u \in H(\Gamma)$ by virtue of Theorem 3.

For basic functions $\{\Psi_k\}$ constructed in Sect. 3 it holds

Theorem 4. *The traces on Γ of the basic functions $\{\Psi_k\}$ form a complete system in $H(\Gamma)$ which is minimal.*

Proof of the completeness in Theorem 4 is based on the approximation theorems by F. Browder [29] for solutions of elliptic PDEs. Theorems [29] can be readily modified to include homogeneous boundary conditions on some part of the boundary.

A Cartesian product $\mathcal{H}(\mathcal{D},\mathcal{N}) \stackrel{\text{def}}{=} \mathring{W}_2^1(\mathcal{D}) \times L_2(\mathcal{N})$ consisting of ordered pairs $\{a_{\mathcal{D}}, a_{\mathcal{N}}\}$, $a_{\mathcal{D}} \in \mathring{W}_2^1(\mathcal{D})$, $a_{\mathcal{N}} \in L_2(\mathcal{N})$, is a Hilbert space with the inner product

$$\big[\{a_{\mathcal{D}}, a_{\mathcal{N}}\}, \{b_{\mathcal{D}}, b_{\mathcal{N}}\}\big]_{\mathcal{H}} = \int_{\mathcal{D}} a_{\mathcal{D}} b_{\mathcal{D}} ds + \int_{\mathcal{D}} (\nabla_\Gamma a_{\mathcal{D}}, \nabla_\Gamma b_{\mathcal{D}}) ds + \int_{\mathcal{N}} a_{\mathcal{N}} b_{\mathcal{N}} ds$$

which induces the norm

$$\|a_{\mathcal{D}}, a_{\mathcal{N}}\|_{\mathcal{H}}^2 = \int_{\mathcal{D}} |a_{\mathcal{D}}|^2 ds + \int_{\mathcal{D}} |\nabla_\Gamma a_{\mathcal{D}}|^2 ds + \int_{\mathcal{N}} |a_{\mathcal{N}}|^2\, ds.$$

Let $L : H(\Gamma) \to \mathcal{H}(\mathcal{D},\mathcal{N})$ be a linear operator defined as $L\psi = \{\psi|_{\mathcal{D}}, \partial_\nu\psi|_{\mathcal{N}}\}\ \forall\, \psi \in H(\Gamma)$. From Theorem 3 it follows

Corollary 1. *The linear operator L is an isometry of $H(\Gamma)$ onto $\mathcal{H}(\mathcal{D},\mathcal{N})$.*

For the basic functions $\{\Psi_k\}$, from Corollary 1 and Theorem 4 follows

Corollary 2. *The system $\{L\,\Psi_k\}$ is complete and minimal in $\mathcal{H}(\mathcal{D},\mathcal{N})$.*

Applying Corollary 2, we approximate the solution $\psi(r,\theta,\varphi)$ of the BVP (2.3)–(2.5) by a sequence $\{\psi^{(N)}(r,\theta,\varphi)\}$ of linear combinations with respect to the first N basic functions Ψ_k:

$$\psi(r,\theta,\varphi) = \lim_{N\to\infty} \psi^{(N)}(r,\theta,\varphi), \qquad \psi^{(N)}(r,\theta,\varphi) = \sum\nolimits_{k=1}^{N} Q_k^{(N)}\, \Psi_k(r,\theta,\varphi). \tag{4.2}$$

Here coefficients $Q_k^{(N)}$ are to be found using the condition of the least square deviation of the approximate solution $\psi^{(N)}$ from the boundary function $h = \{h_{\mathcal{D}}, h_{\mathcal{N}}\} \in \mathcal{H}(\mathcal{D},\mathcal{N})$ corresponding to (2.6) on Γ: $\|L\psi^{(N)} - h\|_{\mathcal{H}} \to \min$. This condition leads to the following system of linear equations with respect to the unknown coefficients $Q_k^{(N)}$, where $l = 1, 2, \ldots, N$:

$$\sum\nolimits_{k=1}^{N} Q_k^{(N)} G_k^l = h^l, \qquad G_k^l = [L\Psi_k, L\Psi_l]_{\mathcal{H}}, \quad h^l = [h, L\Psi_l]_{\mathcal{H}}.$$

The method of least squares guarantees the convergence of the sequence $L\psi^{(N)}$ in the Hilbert space $\mathcal{H}(\mathcal{D},\mathcal{N})$, whence by Corollary 1 follows the convergence of the sequence $\psi^{(N)}$ in the Hilbert space $H(\Gamma)$. Now for the sequence of approximate solutions $\{\psi^{(N)}\}$, reference to Theorem 3 completes the proof of its convergence in $W_2^{3/2}(\Omega)$ to the exact solution $\psi \in \overset{\circ}{W}{}_2^1(\Omega,\gamma)\cap$ $\cap W_2^{3/2}(\Omega)$.

4.3 Asymptotics near the edges

Turn again to the selected edge mentioned in Sect. 4.1. Introduce a cylindrical system of co-ordinates related to this edge by the use of the Cartesian X, Y, Z and the spherical (r, Θ, Φ) co-ordinate systems defined in Sect. 4.1. Namely, let Z be the co-ordinate from the above Cartesian system, Φ the co-ordinate from the above spherical system, and ϱ is defined by the formula $\varrho = \sqrt{r^2 - Z^2}$. Then the desired cylindrical co-ordinate system is (ϱ, Z, Φ).

Starting from the view (4.2) of the solution and using representation (4.1) for the multipoles we derive an asymptotics for the solution of the BVP near the edge with dihedral angle of value $\pi\beta$ when $\varrho \to 0$, $Z \to 0$:

$$\Psi \sim \varrho^{1/\beta}\sin\frac{\Phi}{\beta}\left[\mathcal{I}_{1,1}\, Z^{\mu_1 - 1/\beta} + \ldots\right] + \varrho^{2/\beta}\sin\frac{2\Phi}{\beta}\left[\mathcal{I}_{2,1}\, Z^{\mu_2 - 1/\beta} + \ldots\right] + \ldots .$$

Quantities $\mathcal{I}_{1,1}$ and $\mathcal{I}_{2,1}$ appearing here can be expressed via coefficients of expansions (4.1), (4.2), in particular $\mathcal{I}_{1,1} = 2^{-1/\beta}[\Gamma(1 + 1/\beta)]^{-1} Q_1 D_1^1$, where $\Gamma(x)$ is Gamma-function [27].

Note that coefficients Q_k^n in expansion (4.2) are named intensity factors at the vertex of the cone (polyhedral angle) and quantities $\mathcal{I}_{1,1}$, $\mathcal{I}_{2,1}$ the intensity factors at its edge. From what was said it follows that our method provides computation of all mentioned intensity factors along with the solution itself.

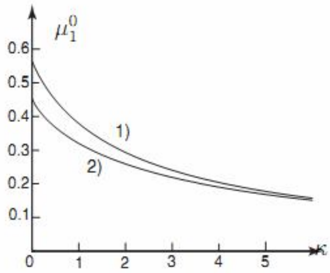

Fig. 1a.

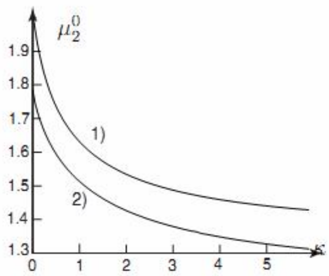

Fig. 1b.

Fig. 1. Dependence of eigenvalues μ_1^0 and μ_2^0 on κ for type I of geometric data and for two variants of parameters: 1) $\alpha = 5/12$, $\delta = 1/6$, $\alpha_{in} = 7/12$, 2) $\alpha = 1/3$, $\delta = 1/6$, $\alpha_{in} = 1/2$.

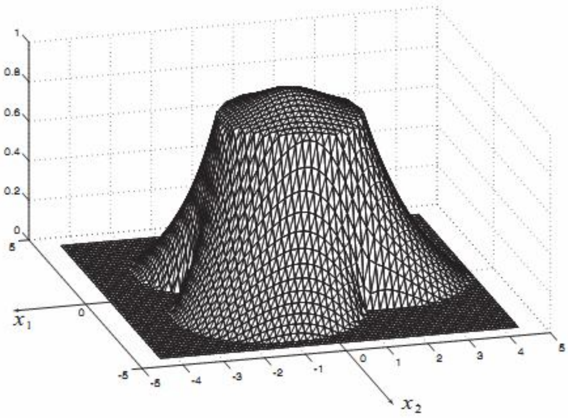

Fig. 2. Space view of the first eigenfunction U_1^{0+} with parameters $\alpha = 5/12$, $\theta_0 = 2/3$, $\kappa = 10$ and eigenvalue $\mu_1^0 = 0.090288$.

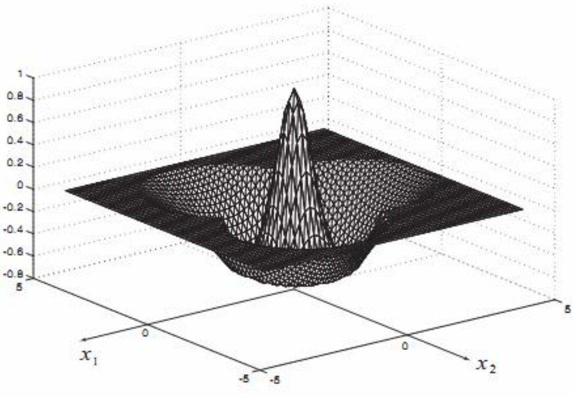

Fig. 3. Space view of the second eigenfunction U_2^{0+} with parameters $\alpha = 5/12$, $\theta_0 = 2/3$, $\kappa = 10$ and eigenvalue $\mu_2^0 = 1.453002$.

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МЕТОД МУЛЬТИПОЛЕЙ ДЛЯ НЕКОТОРЫХ ЭЛЛИПТИЧЕСКИХ КРАЕВЫХ ЗАДАЧ С РАЗРЫВНЫМ КОЭФФИЦИЕНТОМ

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Аннотация. Разработан аналитико-численный метод решения краевых задач в пространственных областях с конусами произвольного основания для эллиптического уравнения с кусочно-постоянным коэффициентом. Решение задачи находится с использованием специальных базисных функций – мультиполей, которые строятся в явном виде. Метод обеспечивает высокоточное вычисление решения, его производных, показателей сингулярности и коэффициентов интенсивности вблизи геометрических особенностей – ребер и вершины конуса.

Ключевые слова: краевые задачи, области с конусами, метод мультиполей, показатели сингулярности, коэффициенты интенсивности.