



STUDY ON THE ACHIEVING TO STRONG COUPLING REGIME FOR InAs/GaAs QUANTUM DOT EMBEDDED IN THE NANOCAVITY

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Abstract. In order to the generation of single photon and the production of dressed states between photons and electrons in nanophotonic structures, achieving strong coupling regime is necessary. One of the best ways achieving to strong coupling, is quantum dots embedded in nanocavity of photonic crystals. In this paper, Hamiltonian of the interaction of nanoparticle - photon, based on semiclassical approach is obtained. Then total Hamiltonian of this system, by Rotating Wave Approximation (RWA) is calculated. By solving this Hamiltonian for Schrödinger equation, eigen values of this system, level shift factor and the strong coupling regime condition for InAs/GaAs quantum dots embedded in the nanocavity was calculated. For three cavities by decay rates of $200\mu\text{eV}$, $600\mu\text{eV}$ and $1000\mu\text{eV}$, the graph $(\Delta E - \gamma_c - g)$ presents that when decreasing decay rate of nanocavity, both coupling constant and levels shift are increased. This means that, only for high quality factor cavities, Strong coupling will occur. Indeed slope of coupling constant graph more than slope of decay rate of cavity. This means that coupling constant plays more important role for achieving to strong coupling regime.

Key words: InAs/GaAs quantum dot, strong coupling regime, nanocavity, dressed state, cavity decay rate.

1. Introduction. A self-assembled single quantum dot (QD) is a nanocrystal made of semiconductor materials that are small enough to exhibit quantum mechanical properties. The electronic properties of these materials are intermediate between those of bulk semiconductors and of discrete molecules. Due to their relatively higher efficiency compared to bulk, they have also found applications as optoelectronic devices [1-5], biotechnology, solar cell [5-12], single photon emitters [13] or qubits for quantum computers [14], quantum interference, Rabi oscillations, Quantum coherence, photon anti-bunching, etc. [15-23]. In zero-dimension structures, the free carriers are confined to a small region by a so called confinement potential providing the quantization of electronic energy states based on the size of the dots. Atom-like discrete energy-levels are occurred when confining the carriers in a nanoregion. The formation of self-assembled InAs QDs on GaAs substrates has attracted much interest, due to their promising applications in nanoscale devices. Various shapes of QDs can be grown by Stranski-Krastanow method [24-26]. The size and shape of such QDs depend on the growth conditions and the used techniques [27-29]. The optical properties of an ensemble of quantum dots are affected by the size distribution and the geometry of the dots [30]. The Stranski-Krastanow method is essentially a self-organized hetero-epitaxial growth during molecular beam epitaxy (MBE) [10]. In this technique, after a number of lattice-mismatched atomic layers deposited on a substrate, accumulated strain energy forces transition from layer to island growth. This happens when a so called wetting layer reaches to a critical 3 thickness of 3-4 nm [31-33]. Semiconductor nanocavity systems with a QD have been investigated because of their unique physics based on cavity quantum electrodynamics and their potentials in future applications such as quantum information processing. In semiconductor microcavity systems [34], vacuum Rabi splitting in the strong coupling regime [35-38] and highly efficient lasing in the weak-coupling regime [39-47] have been observed. Here,



by solving this Hamiltonian for Schrödinger equation, eigenvalues of InAs/GaAs quantum dots embedded in the nanocavity, level shift factor and the strong coupling regime condition for this nanosystem for three cavities by decay rates of $200\mu\text{eV}$, $600\mu\text{eV}$ and $1000\mu\text{eV}$ was studied.

2. Results and Discussion. To simplify the problem, we start the study with the two-level atom system. In quantum mechanics, a two-level system is a system which can exist in any quantum superposition of two independent (physically distinguishable) quantum states. Figure 1 shows the schema of two-level atom system.

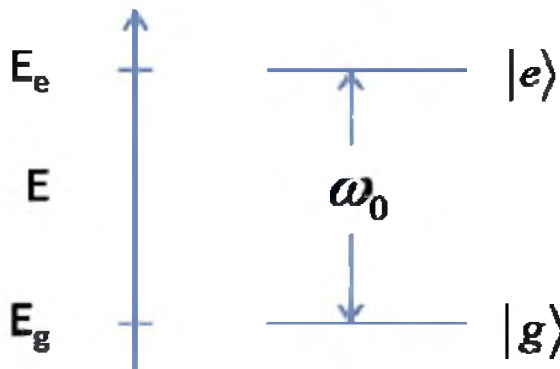


Fig. 1. Schema of two-level atom system.

The Hamiltonian of two-level atom system described by equation (1).

$$H_a = E_g|g\rangle\langle g| + E_e|e\rangle\langle e| \tag{1}$$

By using $|g\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $|e\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and Pauli's matrix $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, equal to equation (2).

$$H_a = \frac{1}{2}\omega_0\sigma_z. \tag{2}$$

By semiclassical approach which in this approach the atoms are classical and the photons are quantum are studied, and by using "rotating wave approximation" (RWA) for this atom equal to equation (3).

$$H_{int} = q\vec{r} \cdot \vec{E}. \tag{3}$$

where q is electric charge ($-e$), \vec{r} is position of electron in the nucleus and \vec{E} is electric field. \vec{r} can be considered as an operator and:

$$q\vec{r} = P_{ee}|e\rangle\langle e| + P_{gg}|g\rangle\langle g| + P_{eg}|e\rangle\langle g| + P_{ge}|g\rangle\langle e|, \tag{4}$$

$$P_{ab} = e\langle a|\vec{r}|b\rangle = \int e\vec{r}^t\Psi_a\Psi_b^*d^3r. \tag{5}$$

This equation for $a = b$ equal to zero and for $a \neq b$ we have $P_{ab} = P_{ba}^*$ with assumption Λ^+ and Λ^-

$$\Lambda^+ = |e\rangle\langle g| = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \tag{6}$$



$$\Lambda^- = |g\rangle\langle e| = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \quad (7)$$

And dipole operator describes by equation(8)

$$e\vec{r} = P\Lambda^+ + P^*\Lambda^-. \quad (8)$$

Interaction Hamiltonian becomes as follows:

$$H_{int} = \hbar(g\Lambda^+ + g^*\Lambda^-)(a - a^\dagger) \quad (9)$$

where in these equations g is coupling constant and is annihilation(creation) operator. By applying operator to ground state atom goes to excited state and by to excited state atom goes to ground state. By using RWA interaction Hamiltonian becomes as follows:

$$H_{int} = \hbar(g\Lambda^+a + g^*\Lambda^-a^\dagger). \quad (10)$$

Finally, total Hamiltonian equal to equation (11),

$$H_{total} = \hbar(g\Lambda^+ + g^*\Lambda^-)(a - a^\dagger) + \hbar\omega a a^\dagger + \frac{1}{2}\hbar\omega_0\sigma_z. \quad (11)$$

By using this Hamiltonian in the time-independent Schrödinger equation Energy Eigenvalues of this system:

$$E_1 = \frac{E_c + E_a}{2} - i\frac{\gamma_c + \gamma_a}{4} + \sqrt{g^2 - \left(\frac{\gamma_c - \gamma_a - 2i\Delta}{4}\right)^2}, \quad (12)$$

$$E_2 = \frac{E_c + E_a}{2} - i\frac{\gamma_c + \gamma_a}{4} - \sqrt{g^2 - \left(\frac{\gamma_c - \gamma_a - 2i\Delta}{4}\right)^2} \quad (13)$$

where in these equations $E_a(E_c)$ is the atom(cavity) energy and γ_a (γ_c) is the atom(cavity) decay rate.

At resonance state which $E_0 = E_a = E_c$

$$E_1 = E_0 - i\frac{\gamma_c + \gamma_a}{4} + \sqrt{g^2 - \left(\frac{\gamma_c - \gamma_a}{4}\right)^2}, \quad (14)$$

$$E_2 = E_0 + i\frac{\gamma_c + \gamma_a}{4} - \sqrt{g^2 - \left(\frac{\gamma_c - \gamma_a}{4}\right)^2}. \quad (15)$$

Here $\Delta E = E_1 - E_2$ which represents the difference between two peaks called Splitting Index:

$$\Delta E = 2\sqrt{g^2 - \left(\frac{\gamma_c - \gamma_a}{4}\right)^2}. \quad (16)$$

The equation (16) expresses coupling regime of system. If $g > |(\gamma_c - \gamma_a)/4|$ there is strong coupling for atom-cavity system. If $g < |(\gamma_c - \gamma_a)/4|$ there is weak coupling for atom-cavity system.

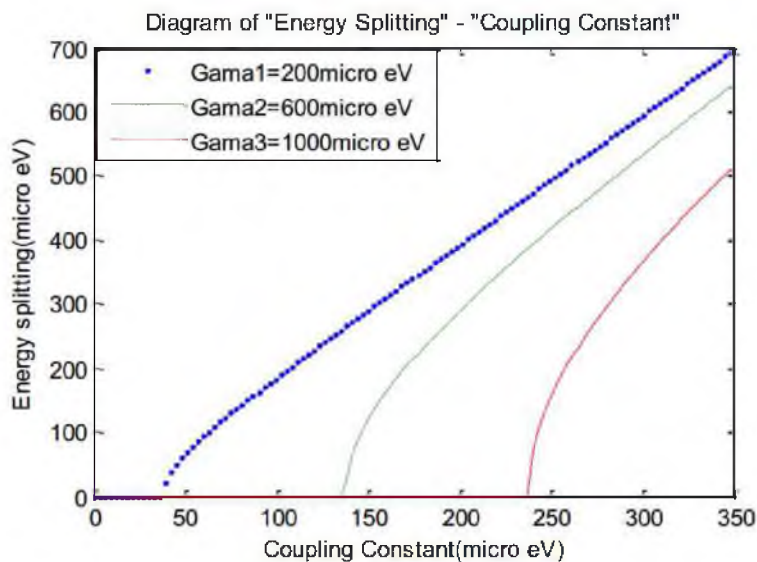


Fig. 2. Graph of coupling constant-energy splitting for three cavities by decay rates of $200\mu\text{eV}$, $600\mu\text{eV}$ and $1000\mu\text{eV}$ of InAs/GaAs quantum dots embedded in the nanocavity.

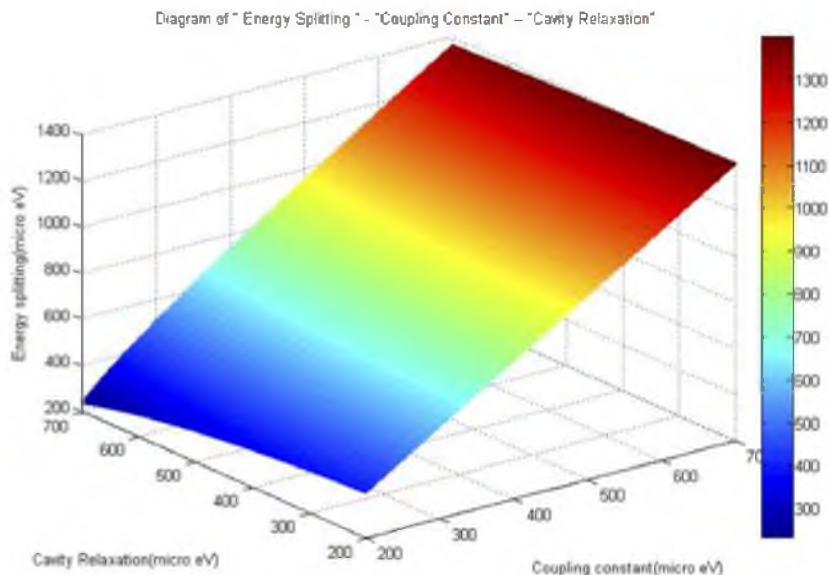


Fig. 3. Graph of coupling constant-cavity relaxation-energy splitting for InAs/GaAs quantum dots embedded in the nanocavity.

Figures 2 and 3 show results of calculation for InAs/GaAs quantum dots embedded in the nanocavity for three cavities by decay rates of $200\mu\text{eV}$, $600\mu\text{eV}$ and $1000\mu\text{eV}$. These figures present that when decreasing decay rate of cavity, both coupling constant and levels shift are increased. This means that, only for high quality factor cavities, Strong coupling will occur. Indeed slope of coupling constant graph more than slope of decay rate of cavity. This means that coupling constant plays more important role for achieving to strong coupling regime.



Conclusion. In this paper by solving the Schrödinger equation, eigenvalues of InAs/GaAs quantum dots embedded in the nanocavity, level shift factor and the strong coupling regime condition for this nanosystem for three decay rates cavities was studied. Results show that when decreasing decay rate of cavity, both coupling constant and levels shift are increased. This means that, only for high quality factor cavities, Strong coupling will occur. Indeed, slope of coupling constant graph more than slope of decay rate of cavity. This means that coupling constant plays more important role for achieving to strong coupling regime.

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