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Smith–Purcell Radiation Driven by the Field of a Standing Laser Wave

A. A. Tishchenko[a, b, *]

[a] *National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Moscow, 115409 Russia*
[b] *Laboratory of Radiation Physics, Belgorod National Research University, Belgorod, 308034 Russia*
**e-mail: tishchenko@mephi.ru*

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Smith–Purcell radiation is well known as a source of quasi-monochromatic electromagnetic radiation that occurs when fast electrons move above a diffraction grating. In this paper, we calculated the Smith–Purcell radiation generation from a flat surface along which there is a field of a standing laser wave. A periodically changing laser field induces a periodic inhomogeneity in the distribution of electrons in the near-surface layer. This periodicity, being an analogue of a diffraction grating, leads to the possibility of generating Smith–Purcell radiation. It is shown that the properties of Smith–Purcell radiation from such an unusual "light" grating are also unusual: the dispersion relation, unlike the standard for Smith–Purcell radiation, does not contain diffraction orders, so that all radiation is concentrated in one peak. The calculated effect makes it possible to control the radiation frequency or angle by changing the laser frequency and may be of interest for the development of new compact radiation sources with tunable characteristics and for non-invasive diagnostics of relativistic electron beams.

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1. INTRODUCTION

Smith–Purcell radiation (SPR) is excited when charged particles move near a diffraction grating. SPR was for the first time experimentally registered in 1953 [1] by American scientists S. Smith and E. Purcell, and bears their name. However, before them, in 1942, this radiation was theoretically predicted by I.M. Frank [2]. Later, SPR has been studied in detail theoretically and experimentally for diffraction gratings of different profiles and from different materials [3–5]. It formed the basis of powerful radiation sources like orotrons (or diffraction radiation generators, in the terminology of V. Shestopalov [6], who devoted considerable time to the development of this device, achieving its practical implementation). At the end of the 20th and the beginning of the 21st century, SPR began to be studied as a basis for non-invasive diagnostics of relativistic electron beams [7–12]. Indeed, as mere the motion of the beam near the grating is sufficient to generate SPR, there is no direct scattering of electrons in the medium. The only thing that can change the characteristics of the electron beam is the recoil momentum, which is for the most part neglected in practice in a relatively soft spectral range. In recent years, SPR has been very actively studied as a source of radiation from more exotic structures: nanoplasmonic crystals [13–17], photonic crystals [18–23], metasurfaces [24–28], SPR from vortex electrons [29–31], SPR near bound states in the continuum [32], etc.

In practice different types of radiation are used for SPR generation: sinusoidal, strip, echelette, etc. All these targets are similar in having the periodic ledges as grating elements. The grating period d defines basic properties of SPR. It is related to the wavelength of radiation λ and the angle of radiation θ, and the velocity of the electron $v = \beta c$ through the formula

$$\lambda s = d(\beta^{-1} - \cos\theta), \tag{1}$$

where s is integer ($s > 0$).

Within the framework of quantum approach, however, to generate radiation by a free electron, it is necessary to transfer a momentum along the electron trajectory to an inhomogeneous structure [33]. Periodically inhomogeneous surface is just one of the special cases of a grating. Meanwhile, periodically inhomogeneous optical properties can be created by the laser field, which keeps the surface flat, but creates optical inhomogeneity of the surface properties. Tuning the laser frequency will make it possible to control the grating period, and thus the characteristics of the radiation.

Below we consider the generation of radiation during the motion of relativistic electrons above a flat surface along which the field of a standing laser wave is formed, see Fig. 1.

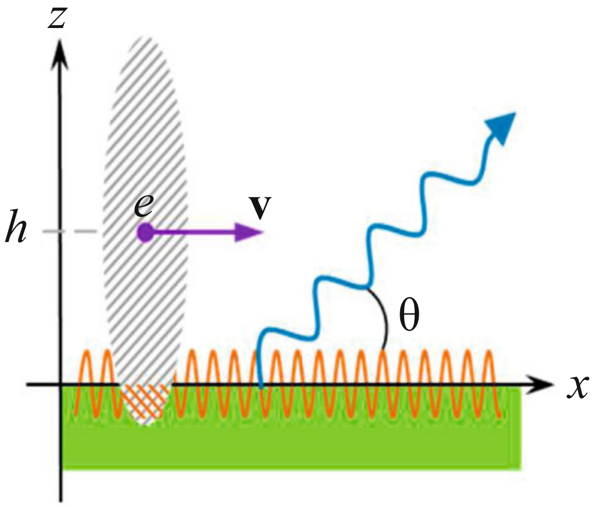

Fig. 1. (Color online) Electron with the charge e (purple) moves at a distance h along the surface of a half-infinite medium (green) and generates SPR from an optical periodic inhomogeneity, induced by the field of a standing laser wave (orange).

2. CURRENT DENSITY AND RADIATION

Let us consider the medium with the function of the permittivity $\varepsilon_0(\omega)$, which is subjected to external field of a standing laser wave

$$\mathbf{E}_L(\mathbf{r},\omega) = \mathbf{E}_L(\omega)\cos(k_0 x), \tag{2}$$

where $k_0 = \sqrt{\varepsilon(\omega_0)}(\omega_0/c)$ is the wavenumber, characterizing the period of the standing wave.

For an isotropic medium, the additional term in the permittivity, which arises due to the additional polarization of the medium by an external field, can only be proportional to the squared field, since there is no constant vector that could be used to compose a tensor of the second rank, linearly proportional to the field [34] (the Kerr effect):

$$\varepsilon_{ij}(\mathbf{r},\omega) = \varepsilon_0(\omega)\delta_{ij} + \chi E_{Li}(\mathbf{r},\omega)E_{Lj}(\mathbf{r},\omega), \tag{3}$$

where χ is a coefficient, characterizing the ability of a medium to polarize under the action of an external field $\mathbf{E}_L$. Neglecting the influence of the anisotropy of the properties of the near-surface layer, we can write the permittivity of the medium in the form

$$\varepsilon(\mathbf{r},\omega) = \varepsilon_0(\omega) + \chi|\mathbf{E}_L(\mathbf{r},\omega)|^2. \tag{4}$$

Thus, considering the familiar formula

$$\varepsilon(\mathbf{r},\omega) = \varepsilon_0(\omega) + \frac{4\pi i}{\omega}\sigma(\mathbf{r},\omega), \tag{5}$$

one can see, that a standing wave of a laser directed along the surface creates in the near-surface layer periodically modulated optical properties described by the function $\sigma(\mathbf{r},\omega)$, which can be found by comparing Eqs. (4) and (5):

$$\sigma(\mathbf{r},\omega) = \frac{\omega}{4\pi i}\chi|\mathbf{E}_L(\omega)|^2\cos^2(k_0 x). \tag{6}$$

Designating $I_L = |\mathbf{E}_L(\omega)|^2$, we can find that the current density

$$\mathbf{j}(\mathbf{r},\omega) = \frac{\omega}{4\pi i}\chi I_L\cos^2(k_0 x)\mathbf{E}_0(\mathbf{r},\omega) \tag{7}$$

arises under the action of an external field $\mathbf{E}_0(\mathbf{r},\omega)$ of free electrons, moving at a distance h along the surface of the medium.

Assuming that this near-surface layer with the periodic inhomogeneity is thin, we can calculate the characteristics of radiation propagating in vacuum. The Fourier transform of Maxwell's equations is algebraic, which makes it possible to obtain an equation relating the components of the Fourier transforms of the field $\mathbf{E}(\mathbf{q},\omega)$ and $\mathbf{j}(\mathbf{q},\omega)$:

$$A_{ij}E_j(\mathbf{q},\omega) = -\frac{4\pi i\omega}{c^2}j_i(\mathbf{q},\omega), \tag{8}$$

where $A_{ij} = q_i q_j - (q^2 - \omega^2/c^2)\delta_{ij}$. The inverse tensor must have the same tensor structure as the direct one, i.e., has to look like $A_{ij}^{-1} = aq_iq_j + b\delta_{ij}$. Unknown coefficients a and b can be found from the condition $A_{ij}^{-1}A_{js} = \delta_{is}$. As a result, the Fourier transform of the field reads

$$E_i(\mathbf{r},\omega) = \frac{4\pi i}{\omega}\int d^3q e^{i\mathbf{qr}}\frac{k^2\delta_{ij} - q_iq_j}{q^2 - k^2}j_j(\mathbf{q},\omega), \tag{9}$$

where $k = \omega/c$. To calculate the radiation field, it is convenient to use the well-known asymptotic formula

$$\int d^3q e^{i\mathbf{qr}}\frac{f(\mathbf{q})}{q^2 - k^2 - i0}\xrightarrow{kr\gg 1}2\pi^2\frac{e^{ikr}}{r}f(\mathbf{k}), \tag{10}$$

where $\mathbf{k} = k\mathbf{r}/r$. Then the distribution over angles and frequencies of the energy radiated by the charge is

$$\frac{d^2W(\mathbf{n},\omega)}{d\Omega d\omega} = \frac{e^2}{c}\frac{\chi^2 I_L^2}{\omega^2 4\pi^4}|\mathbf{k}\times\xi|^2, \tag{11}$$

where

$$\xi = \int d^3r e^{i\mathbf{kr}}\cos^2(k_0x) \times\int d^3q e^{i\mathbf{qr}}e^{-iq_zh}\frac{k^2\mathbf{v} - \omega\mathbf{q}}{q^2 - k^2}\delta(\omega - \mathbf{qv}). \tag{12}$$

Let a free electron move along the axis x. Calculating the integrals in Eq. (12) we find

$$\frac{d^2W(\mathbf{n},\omega)}{d\Omega d\omega} = \frac{e^2}{c}\chi^2 I_L^2 \frac{\pi}{8}\frac{T}{(1-\beta n_x)^2}e^{-h\rho} \times \frac{|\mathbf{k}\times(\mathbf{b}-i\omega\rho\mathbf{e}_z)|^2}{\omega^2\rho^2(\rho^2+k_z^2)}\delta\left(\omega - \frac{2\omega_0\sqrt{\varepsilon(\omega_0)}}{\beta^{-1}-n_x}\right), \quad (13)$$

where γ is a Lorentz factor of an electron, $\mathbf{n}$ is the unit wave-vector, and

$$\mathbf{b} = \frac{\omega^2}{v\gamma^2}\mathbf{e}_x + \omega k_y \mathbf{e}_y, \quad \rho = \frac{\omega}{c\beta\gamma}\sqrt{1+\beta^2\gamma^2 n_y^2}. \quad (14)$$

While obtaining Eq. (13) we took into account that the Cherenkov radiation cannot be generated in vacuum, and that the frequency ω is positive. Also, we used the known representation of the squared delta function as a product of the delta function and the time of the radiation process T, like it was done by Tamm in the problem of Cherenkov radiation at a finite time of flight.

3. ANALYSIS

Equation (13) contains the exponent $e^{-h\rho}$ usual for any kind of diffraction radiation or SPR. The fact that the distribution of radiated energy over angles and frequencies is proportional to squared intensity of the laser field I_L^2 is also expected: this is a characteristic feature for isotropic materials, whose optical properties, being induced by an external field, are characterized by a squared field $|\mathbf{E}_L|^2$ according to the Kerr law. The presence of a squared field in the density of the induced current (see Eq. (7)), leads to a quadratic dependence of the radiation intensity on the intensity of the laser wave field in Eq. (13). Also, a linear dependence of the distribution of radiated energy over angles and frequencies on the time T of electron flight above the surface where a standing wave exists is also expected. This is a characteristic feature for all types of radiation generated from a unit path length (like Cherenkov radiation, SPR, etc.).

Unusual in Eq. (13) is the argument of the delta function, which relates the radiation frequency to the grating angle and the grating period:

$$\omega = \frac{2\omega_0\sqrt{\varepsilon(\omega_0)}}{\beta^{-1}-n_x}. \quad (15)$$

The presence of the factor two in the denominator of the right side stems from the quadratic dependence on the amplitude of the external field in the Kerr law; in a similar fashion, the second-harmonic generation occurs in the nonlinear optics. Let us rewrite (15) as

$$\lambda = \frac{\lambda_0}{2}(\beta^{-1} - \cos\theta). \quad (16)$$

The parameter $\lambda_0 = 2\pi c/(\omega_0\sqrt{\varepsilon(\omega_0)})$ determines the period of a standing wave in a medium with permittivity $\varepsilon(\omega_0)$. Equation (16) differs from Eq. (1) by the absence of the diffraction orders. This fundamental difference is a consequence of the fact that the surface is flat. It was shown in [12] that the diffraction orders emerge in the dispersion relation for any profile of a periodically inhomogeneous surface.

It follows from Eq. (16) that the minimal wavelength $\lambda_{\min} = \lambda_0/(4\gamma^2)$ depends on the Lorentz factor of an electron. This dependence is a feature of the inverse Thomson (or, in the quantum mode, Compton) scattering. This can be explained by the fact that the dynamic polarization of the medium, which is a source of radiation, is created by the Lorentz-deformed Coulomb field of a fast electron (see the intersection of gray and orange areas in Fig. 1). For the relativistic electrons, this region moves with a relativistic velocity and interacts with the field of a (standing) laser wave. In this formulation the problem has the characteristic features of the problem of the inverse Thomson/Compton scattering of photons on relativistic electrons.

Note that the use of a laser in such problems is not something exotic. For example, in the well-developed method of electro-optic sampling, which has a very wide application [35, 36], including the diagnostics of electron beams [37], the fast free electrons' field perturbs the electron density in surface layer, thereby changing its optical properties, which is recorded by a laser beam directed along the surface.

The effect predicted in this work can be used in the development of non-invasive schemes for diagnostics of the relativistic electron beams, or for modifying an electro-optical scheme. In this regard, the sharp quadratic dependence of the radiation intensity on the intensity of the laser wave according to Eq. (13) can be especially attractive from the experimental point of view.

The obtained results mean that, in contrast to the conventional SPR from a grating, the studied type of radiation does not contain diffraction orders that define the change in the propagation direction by the radiation of fixed wavelength. In the considered case, all radiation at one frequency is concentrated near one direction. This effect allows controlling the frequency or angle of radiation by changing the frequency of the laser, and can form the basis of new compact radiation sources with tunable characteristics.

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CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

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