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Motion of Grinding Bodies within the Chambers of a Centrifugal Mill

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Abstract—The motion of grinding bodies within the chambers of a centrifugal mill is studied theoretically. Analytical formulas permit harmonization of the ball motion in different mill chambers.

Keywords: mill, lever mechanism, grinding chambers, grinding bodies, coordinate system, angular velocity, shaft speed

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INTRODUCTION

Analysis of equipment for fine and superfine grinding shows that the motion of the grinding chamber is the same in all mills, for all stages of grinding [1–4]. The grinding efficiency can only be altered by changing the shape and size of the grinding bodies, the number employed, and the chamber length.

Accordingly, it is obvious that the best organization of the grinding process within a single machine involves chamber trajectories appropriate to the operational conditions: intense impact load and partial abrasion for coarse grinding; impact loading with greater abrasion for fine grinding; and intense abrasion for superfine grinding.

A centrifugal grinding system with different chamber trajectories permits the transition from intense impact loading to intense abrasion [5, 6]. Intense impact loading corresponds to linear motion of the grinding chamber, while intense abrasion corresponds to chamber rotation.

The motion of grinding bodies in individual mill chambers was considered in [7–10]. Research shows that the motion of the charge in each chamber corresponds to different speeds of the eccentric shaft. Thus, the range of shaft speeds in which each chamber operates must be determined. In particular the motion of the grinding bodies in the upper and lower chambers must be harmonized. The upper chamber perform reciprocating motion in a vertical plane, which results in impact of the bodies on the charge; the rotary motion of the lower chamber ensures abrasive action of the grinding bodies on the charge.

ANALYSIS

We assume that, in the upper grinding chamber, the grinding ball is continuously tossed upward, with a single impact. In other words, after impact with the lower chamber wall, the ball moves toward the upper wall, but without impact. This may be explained in that, with the specified parameters of the experimental centrifugal grinding system ($e = 0.02$ m, $\nu = 33$), the centrifugal forces arising in circular motion of the balls in the lower chamber increase considerably when the crankshaft speed is increased to a value corresponding to two impacts. Note that the material to be ground moves together with the grinding balls. That considerably decreases the grinding efficiency.

In the upper chambers, performing reciprocating motion, the kinetic energy T_1 (J) is given by the expression

$$T_1 = \Delta T_1 + T_1',$$

where T_1' is the kinetic energy of linear ball motion, J (Fig. 1); and ΔT_1 is the kinetic energy transmitted to the charge (volume V) for its disintegration, J.

We may determine T_1' from the formula

$$T_1' = \frac{M_1 v_{S1}^2}{2}, \tag{1}$$

where M_1 is the total mass of the grinding balls in the chamber, kg; and v_{S1} is the center-of-mass velocity of the grinding balls in the coordinate system associated with the chamber, m/s.

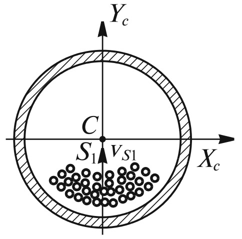


Fig. 1. Motion of grinding bodies in upper chambers.

The counterpart of the center-of-mass velocity of the grinding balls in the upper chamber is given by the following equation, according to [3, 4]

$$v_{S1}(\varphi) = -eR\left(\sin\varphi_0 - \frac{\sin\varphi_0\cos\varphi_0}{\sqrt{\nu^2 - \sin^2\varphi_0}}\right), \tag{2}$$

where φ_0 is the rotational angle of the eccentric shaft, deg; e is the eccentricity of the shaft, m; R is the recovery of the velocity after impact (in practice, $0 \le R < 1$); and ν is the relative length of the connecting rod.

According to experimental data regarding ball motion in different chambers, $\omega_{\min}$ is due to the onset of working conditions in the upper chambers. Hence, to determine $\omega_{\min}$, we simply consider the condition $T_1' = \Delta T_1$.

From Eq. (1)

$$\frac{M_1 v_{S1}^2}{2} = \Delta T_1.$$

Hence

$$v_{S1} = \sqrt{\frac{2\Delta T_1}{M_1}}.$$

The velocity v_{S1} and its counterpart $v_{S1}(\varphi)$ are related as follows

$$v_{S1} = v_{S1}(\varphi)\frac{d\varphi_0}{dt} = v_{S1}(\varphi)\omega.$$

Therefore

$$\omega = \frac{v_{S1}}{v_{S1}(\varphi)}.$$

Determining $v_{S1}(\varphi)$ from Eq. (2), we find that

$$\omega = \frac{v_{S1}}{eR\left(\sin\varphi_0 - \frac{\sin\varphi_0\cos\varphi_0}{\sqrt{\nu^2 - \sin^2\varphi_0}}\right)}.$$

A minimum absolute value $\omega_{\min}$ at constant e and R is possible with maximum absolute value of the expression in parentheses. If $\varphi_0 = \pi/2$, we obtain a value of 1; if $\varphi_0 = -\pi/2$, we obtain a value of −1.

Thus

$$\omega_{\min} = \left|\frac{v_{S1}}{eR}\right|. \tag{3}$$

Adopting grinding conditions in the upper chamber on the basis of the experimental results (S_m = 400 m^2/kg, S_0 = 30 m^2/kg, S = 150 m^2/kg), we determine the minimum energy ε (J/m^3) that must be supplied to unit volume of the material to be ground in accordance with the recommendations in [11, 12]. We find that $\varepsilon_1 = 45.3 \times 10^6$ J/m^3. The volume of the material in a single chamber is $V_1 = 1.5 \times 10^{-3}$ m^3.

From the formula

$$\varepsilon = \frac{Pt}{V} = \frac{\Delta A}{V} = \frac{\Delta T}{V},$$

we find the minimum kinetic energy $\Delta T_1 = 67.9 \times 10^3$ J required to grind material of volume V_1. To grind that material from specific surface S_0 to S requires a certain number of impact cycles over a time Δt. Given the design productivity of the mill (Q = 200 kg/h), the time for the material to pass through a single chamber is $\Delta t \approx 440$ s, if its density is ρ = 2600–2700 kg/m^3. On that basis, the power required in grinding is $P_1 = \Delta T_1/\Delta t = 67.9 \times 10^3/440 = 154.3$ W.

On the other hand, the power may also be determined from the impact force F_1 (N) due to the action of the chamber wall on the set of grinding bodies

$$P_1 = F_1 v_{S1}. \tag{4}$$

The impact force is determined on the basis that the impact momentum is equal to the sum of the momenta of the grinding bodies

$$\bar{F}_1\tau = \bar{Q}_1 = \sum_1^n m_k \bar{v}_k = M_1 \bar{v}_{S1}, \tag{5}$$

where τ is the impact duration, s.

Expressing Eq. (5) in terms of F_1 and substituting the result into Eq. (4), we obtain

$$P_1 = \frac{M_1 v_{S1}^2}{\tau}.$$

Hence

$$v_{S1} = \sqrt{\frac{P_1\tau}{M_1}}.$$

To determine the duration of impact, we use a formula obtained in studying the transverse impact of a solid on a surface [13, 14]

$$\tau = \pi\sqrt{\frac{ml^3}{48EJ}},$$

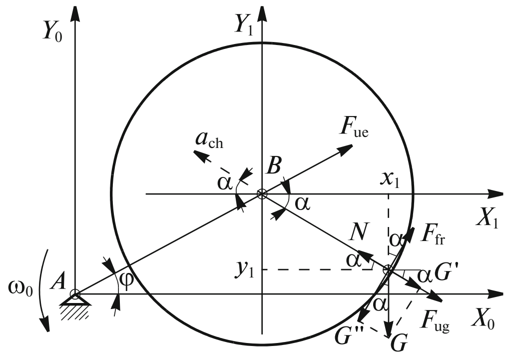
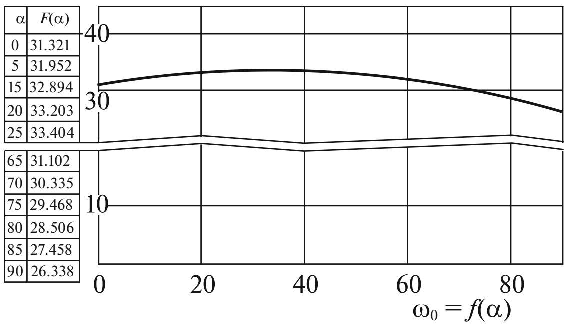


Fig. 2. Configuration of lower grinding chamber.



α	F(α)
0	31.321
5	31.952
15	32.894
20	33.203
25	33.404
65	31.102
70	30.335
75	29.468
80	28.506
85	27.458
90	26.338

Fig. 3. Dependence of ω_0 on α.

where $m = M_1$ is the mass of the solid, kg; l is the chamber length, m; $E = 2 \times 10^{11}$ Pa is the elastic modulus of the chamber wall; and $J = 0.75 \times 10^{-5}$ m^4 is the moment of inertia of the chamber cross section.

When the mass of the grinding bodies within the chamber is $M_1 \approx 12$ kg, we find that $\tau = 0.5 \times 10^{-3}$ s. Taking into account that the chamber contains charge to be ground, we assume that $\tau \approx 0.001$ s.

From Eq. (3), we calculate the velocity $v_{S1} = 0.113$ m/s. When the eccentricity of the shafts $e = 0.02$ m and the recovery $R = 0.5$, we obtain the minimal speed of the eccentric shaft $\omega_{\min} = 17.1$ s^{-1}.

Theoretical analysis yields the dependence of the speed of the eccentric shaft ω_0 on the angular velocity ω_b at which the grinding ball of mass m rolls over the wall of the lower grinding chamber relative to the center of the chamber's coordinate system (Fig. 2)

$$\omega_0 = \sqrt[4]{\frac{\omega_b^4 {R_B}^2\left[(\cos\alpha + f\sin\alpha)^2 + (f\cos\alpha - \sin\alpha)^2\right] + 2\omega_b^2 R_B\left[(\cos\alpha + f\sin\alpha)A + (f\cos\alpha - \sin\alpha)B\right] + A^2 + B^2}{e^2}}, \quad (6)$$

where

$$A = gf\sin^2\alpha - g\sin\alpha\cos\alpha;$$

$$B = gf\sin\alpha\cos\alpha + g\sin^2\alpha - g;$$

α is the angle defining the ball's position, deg; g is the acceleration due to gravity ($g = 9.8$ m/s^2); and R_B is the distance from the coordinate origin (point B) to the ball's center of mass ($R_B = 0.075$ m).

On the basis of Eq. (6), we plot the dependence of ω_0 on the angle α (Fig. 3). By that means, we may find the value of $\omega_0(F(\alpha))$ that corresponds to the value of α in the lower grinding chamber ensuring the required ball position and hence to the required operating conditions. The angle α is measured from the horizontal axis X_1.

We know that cascade motion of the grinding balls sets in when $\alpha \approx 10°$, while waterfall motion begins at about 26° [15, 16] When $\alpha = 90°$, the motion of the balls is centrifugal. In other words, they do not break away from the inner surface of the chamber.

It is evident from Fig. 3 that, at the value $\omega_0 = \omega_{\min} = 17.1$ s^{-1} ensuring single-impact operation of the grinding balls in the upper chamber, the grinding balls in the lower chamber cannot reach the horizontal axis. In other words, not even cascade motion of the balls is possible. Hence, the grinding efficiency in the lower chamber will be low. Maximum efficiency in the lower chamber entails centrifugal motion of the grinding balls, with $\alpha = 90°$. In that case, the speed of the eccentric shaft must be no less than $\omega_0 = 34$ s^{-1}.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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